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NOTE

A novel image quality index using Moran *I* statistics

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Abstract

Measurement of image quality is very important for various applications such as image compression, restoration and enhancement. Conventional methods (e.g., mean squared error; MSE) use error summation to measure quality change pixel by pixel and do not correlate well with subjective quality measurement. This is due to the fact that human eyes extract structural information from the viewing field. In this study a new quality index using a Moran *I* statistics is proposed. The Moran statistic that measures the sharpness from a local area is a good index of quality as most image processing techniques alter the smoothness of the image. Preliminary results show that the new quality index outperforms the MSE significantly under various types of image distortions.

1. Introduction

Measurement of image quality is very important for various image processing applications such as compression, restoration, enhancement and reproduction (Okkalides and Efremides 1994, Good *et al* 1994, Eskicioglu and Fisher 1995, Cosman *et al* 2000, Burgul *et al* 2000, Avcibas *et al* 2002). Human observer studies have been conducted to assess the changes in image quality. The receiver operating characteristic (ROC) analysis is the dominant technique for evaluating image quality. A subjective image quality index can be evaluated from the area under the ROC curves. In an ROC study (Wong *et al* 1995) for a specific task application, the image observers are asked to review the processed images with or without an abnormality to provide a binary decision along with their degree of certainty. The diagnostic accuracies of these images are then compared with that of the original images. The ROC analyses are expensive and time consuming. A typical ROC study would require more than 300 images to obtain a statistically significant result (Wong *et al* 1995).

Objective evaluations of image quality are attractive because they are easy to calculate and are independent of viewing conditions and individual observers. The mean squared error

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(MSE) is most commonly used to measure the quality changes of images objectively. In fact, the FDA guidance document for picture archiving and communication systems (PACS) requires the manufacturers to report the MSE of their lossy compression techniques (Wong *et al* 1995). The MSE measures the quality change by taking the mean of the squared differences between all corresponding pixels in the original and processed images. The MSE is sensitive to degradation. However, MSE neither provides any information regarding the type of loss that causes the quality deterioration nor correlates well with subjective quality measurement. The problem of MSE is due to the fact that it only calculates the sum of error between corresponding pixels. But the human eyes extract structural information based on the relative distribution of grey levels in the neighbouring pixels. The structural information is not affected by the magnitude of change between the images.

In our opinion, a good quality index for the comparison between two images should be: (1) extracted from structural information; (2) calculated on a small region such that small variation can be detected; and (3) based on regional grey level distribution. It is usually desired to evaluate the entire image using a single quality value although the image quality is often space variant. Therefore, it is practical to measure the quality index locally and then combine them together. In this study, we propose the use of a Moran *I* statistics (Cliff and Ord 1981) calculated on a sliding window as the quality index.

In this note, we first introduce the conventional MSE index, a viewing area based Q quality index, and the proposed quality index. Then the results of the comparison among different quality indexes when applied to various processed images are presented. Finally, the advantages of the proposed index are discussed.

2. Objective image quality measurement

In the following discussion, let **G** and **H** represent the original and processed images and their pixel values are denoted by f_G and f_H, respectively.

2.1. Pixelwise error based measurement

This class of methods measures the quality degradation in the form of a Minkowski metric

$$E = \frac{1}{M}\left[\sum_i |f_G(i) - f_H(i)|^\beta\right]^{1/\beta} \tag{1}$$

where M is the total number of pixels and β is a constant. Among them, MSE is the most common criterion used. It measures the image difference by taking the mean of the squared differences between all corresponding pixels. It is very sensitive to the image degradation but is completely non-specific and it does not correlate well with subjective quality measures. For example, when two images are relatively displaced by one pixel, the image quality is the same but the measured MSE is very large.

2.2. Q index

To avoid the difficulties encountered by MSE, a Q index has been proposed by Wang and Bovik (2002). It estimates a quality index from a local region. It is defined as

$$Q = \frac{4\sigma_{GH}\bar{f}_G\bar{f}_H}{\left(\sigma_G^2 + \sigma_H^2\right)\left(\bar{f}_G^2 + \bar{f}_H^2\right)} \tag{2}$$

where $\bar{f}$ and σ^2 are the mean and variance of the pixel values inside the window and $\sigma_{\mathrm{GH}} = \frac{1}{N-1}\sum_{i=1}^{N}[f_{\mathrm{G}}(i) - \bar{f}_{\mathrm{G}}][(f_{\mathrm{H}}(i) - \bar{f}_{\mathrm{H}}]$ is the covariance between images **G** and **H**. The dynamic range of Q is $[-1, 1]$ with the best value of 1 being achieved when **G** and **H** are identical. Note that the covariance measurement is dependent upon the relative location between sequential pixels. The Q index is calculated for a window size of 8×8 using a sliding window approach without overlapping.

2.3. Moran I test

The Moran coefficient I (Chuang and Huang 1992) for pixels in an $r \times c$ window is calculated as

$$I = \frac{\sum_{j=1}^{r\times c}\sum_{i=1}^{r\times c}\delta_{ij}[f(i) - \bar{f}][f(j) - \bar{f}]/S_0}{\sum_{i=1}^{r\times c}[f(i) - \bar{f}]^2/N} \tag{3}$$

where $f(i)$ is the grey level of pixel i, $\bar{f}$ is the mean grey level inside the window, $\delta_{ij} = 1$ if pixel i and j are adjacent, and 0 otherwise, $S_0 = \sum\sum\delta_{ij}$ is the number of contiguous pairs (equal to $4rc - 2r - 2c$ for a rectangular lattice) and N $(= rc)$ is the total number of pixels. This I value measures the unsharpness of the region under study. For a smooth region, the grey levels of adjacent pixels are more or less the same, the calculated I is large. Note that $I = 1$ when all pixels have the same grey levels. If the pixels inside the window are randomly distributed, the random variable I can be approximated by a normal distribution (when N is large enough) with mean and variance given by

$$m = -1/(N - 1) \tag{4}$$

and

$$\sigma^2 = \frac{N\big[(N^2 - 3N + 3)S_1 - NS_2 + 3S_0^2\big] - K\big[N(N-1)S_1 - 2NS_2 + 6S_0^2\big]}{(N-1)(N-2)(N-3)} - m^2 \tag{5}$$

where $K = N\sum[f(i) - \bar{f}]^4\big/\big[\sum(f(i) - \bar{f})^2\big]^2$, $S_1 = 2S_0$, and $S_2 = 8(8rc - 7r - 7c + 4)$. The standardized normal statistic

$$z = \frac{I - m}{\sigma} \tag{6}$$

is often employed. Since sharpness is an important parameter of image quality, the z statistic can serve as the quality index for pixels inside the window.

The proposed quality index is defined as the difference or the squared difference between the z values of two corresponding windows. The mean Moran error (MME) and mean squared Moran error (MSME) are the index average of all windows, i.e.

$$\mathrm{MME} = \frac{rc}{M\sum\bar{f}_{\mathrm{G}}}\sum(z_{\mathrm{G}} - z_{\mathrm{H}})\bar{f}_{\mathrm{G}} \tag{7}$$

$$\mathrm{MSME} = \frac{rc}{M\sum\bar{f}_{\mathrm{G}}}\sum(z_{\mathrm{G}} - z_{\mathrm{H}})^2\bar{f}_{\mathrm{G}}. \tag{8}$$

Since most background areas are associated with lower grey levels they should have smaller weights in the perceived image quality. Therefore, we include $\bar{f}_{\mathrm{G}}$ in the calculation so that regions with higher grey levels will have more weights for quality index measurement. The $r \times c$ $(= 8 \times 8)$ window size should be large enough to be statistically significant and yet small enough to be sensitive to the local difference between images. Note that MME is positive

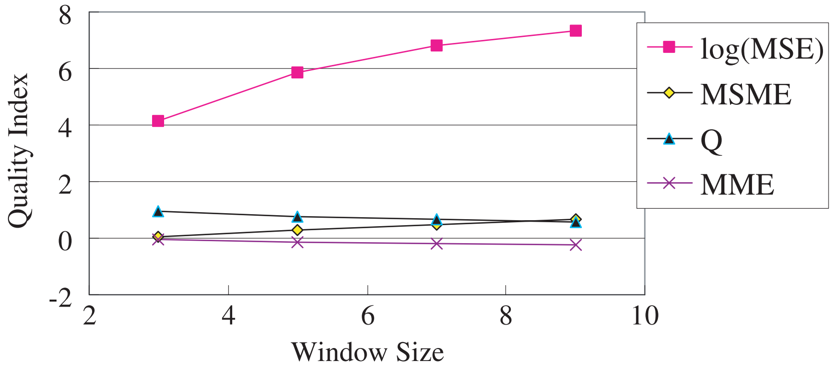

Figure 1. The quality index measured on an MR image processed by a median filter with various window sizes. (The log scale is used for MSE for better visualization.)

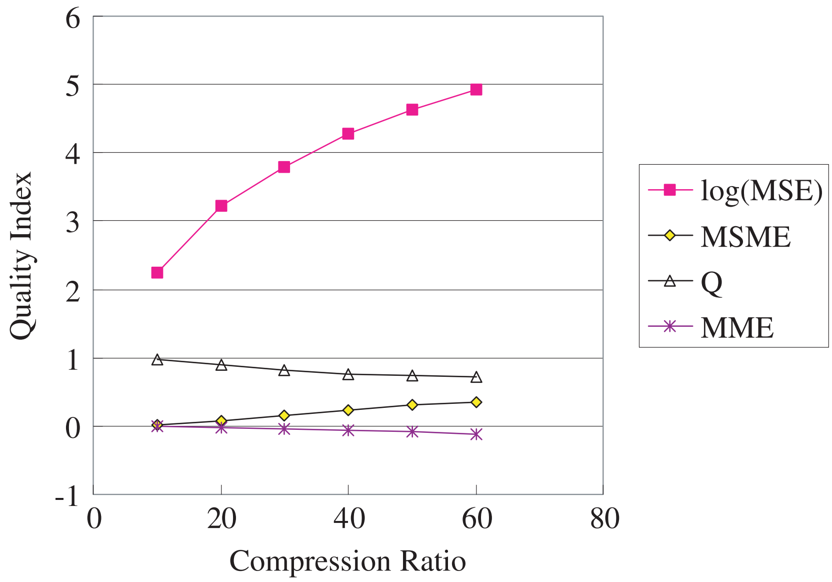

Figure 2. The quality index measured on the reconstructed MR image from a lossy wavelet compression with various compression ratios. (The log scale is used for MSE for better visualization.)

(i.e., $z_G > z_H$) if the processed image (**H**) is becoming sharper or noisier than the original image (**G**) and negative if the processed image is smoother.

3. Results

We use an MR image (512×512, 12 bits) with different types of distortion to test the proposed method and compare the results with the MSE and Q index. Figure 1 shows the quality index measured on the MR image processed with a median filter with various window sizes. Figure 2 shows the quality index measured on wavelet compressed images (Pegasus Imaging Corp., FL, USA) of various compression ratios. In figure 3, we randomly assign '0' or '1' to

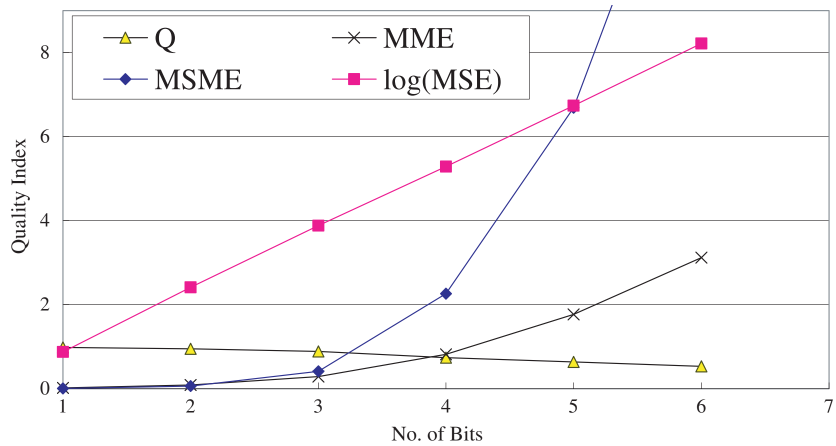

Figure 3. The quality index measured on the MR image as a function of number of lower n (= 1 to 6) bits data that are randomly manipulated. (The log scale is used for MSE for better visualization.)

the lower n (= 1 to 6) bits data of the MR image and measure their quality indexes. These results show that both MSME and MME correlate well with the MSE and the Q index. It can be seen from these figures that the Q index is less sensitive to the quality change. This is due to the fact that the Q index (Wang and Bovik 2002) measures the 'ratio' of three combining factors (correlation coefficient, mean luminance and contrast) between images and in most cases the relative change of these factors is small. The MSE, MSME and MME measure the 'error' between images and are more sensitive to quality change. In figures 1 and 2, the images become smoother after median filtering and compression, and the MME are negative, while in figure 3 the image becomes noisier after bit manipulation and the MME value is positive.

The quality index must be applicable to various image processing applications and be able to provide meaningful comparison across different types of image distortions. In the following, we compare the quality index under various corruptions. The 'Lena' image is employed for demonstration (figure 4). Although it is not related to medical images, the Lena image is widely employed in the image processing field. The images are arranged with increasing MSME (deteriorating quality). The overall quality indexes are tabulated in table 1. The performance of MSE is poor in the sense that the measured value changes significantly (e.g., figures 4(b) and (c)) while the image quality is only modified slightly. Another example (not shown here) is that a constant shift of the grey level to the image will cause a large change in MSE value while the Q index and Moran index remain almost the same. This is due to the fact that MSE is sensitive to the grey level difference between corresponding pixels. Both Q index and Moran index measure the structural distortions and are insensitive to the mean shift in grey level.

In general, the MSME (MME) and Q index have shown good correlation in the quality evaluation (refer to table 1) except for figures 4(a) and (b). The Q index is sensitive to the displacement of image (figure 4(b)) due to the covariance measurement. The Q index is rather small for an image processed by histogram equalization. Histogram equalization is an image enhancement process, which is supposed to increase the image quality. The quality index should not be too different from the ideal one. The decrease in Q index is due to the large grey level shift after equalization. The MSME (MME) does not show much change in the quality

Figure 4. Evaluation of test image processed by various techniques. (a) Histogram equalization; (b) spatial displacement of (1,1); (c) window/level; (d) lower 3 bits manipulation; (e) median filter with window size of 3×3; (f) wavelet compression with compression ratio of 20.

index for this image. Another large discrepancy between MSME (MME) and Q index is the image processed by the window/level technique. In this enhancement processing, the grey level does not significantly change and the Q index remains fairly high. However, the Moran test is sensitive to the grey level overflowed regions in the image and yields a slightly large MSME (MME) index.

Table 1. Quality measurement of 'Lena' image with various effects.

Image	Processing type	MSE	Q	MSME	MME
Figure 4(a)	Histogram equalization	1144.2	0.74	0.065	0.056
Figure 4(b)	Spatial displacement	141.2	0.5	0.95	0.03
Figure 4(c)	Window/level	66.1	0.96	1.41	0.32
Figure 4(d)	Lower 3 bits manipulation	9.5	0.79	3.77	1.39
Figure 4(e)	Median filter	14.47	0.78	7.7	−2.06
Figure 4(f)	Wavelet compression	16.03	0.68	11.37	−2.39

4. Discussion and conclusion

The use of Moran statistics as a quality index has several advantages: (1) it measures the sharpness of image that is strongly related to image quality; (2) it is sensitive to the quality change; (3) it is a regional measurement and is relatively unaffected by the spatial displacement between two images; (4) it measures the structural distortion and not pixel variation; (5) it measures the overall quality change and yet is sensitive to local variation; (6) the sign (positive or negative) of MME is an indication of the type (sharpening or smoothing) of quality change.

5. Summary

In this note, we propose a new image quality measurement based on Moran *I* statistics of a viewing field. The quality index is applied to various processed images and the measured values correlate well with the degree of quality degradation. This index can be used to specify the types of quality change. The future work is to apply this method to *blind quality measurement*, i.e., to assign quality indexes that are consistent with human perception without explicit comparison with the reference image.

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