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RESEARCH ARTICLE

Exponential synchronization of memristive Cohen–Grossberg neural networks with mixed delays

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Abstract This paper concerns the problem of global exponential synchronization for a class of memristor-based Cohen–Grossberg neural networks with time-varying discrete delays and unbounded distributed delays. The drive-response set is discussed. A novel controller is designed such that the response (slave) system can be controlled to synchronize with the drive (master) system. Through a nonlinear transformation, we get an alternative system from the considered memristor-based Cohen–Grossberg neural networks. By investigating the global exponential synchronization of the alternative system, we obtain the corresponding synchronization criteria of the considered memristor-based Cohen–Grossberg neural networks. Moreover, the conditions established in this paper are easy to be verified and improve the conditions derived in most of existing papers concerning stability and synchronization for memristor-based neural networks. Numerical simulations are given to show the effectiveness of the theoretical results.

Keywords Exponential synchronization · Memristor · Cohen–Grossberg neural networks · Unbounded distributed delay · Control

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Introduction

Memristor is a contraction for memory resistor, which was firstly postulated by Chua (1971) and Chua and Kang (1976). However, memristor did not cause much attention of researchers until the papers Strukov et al. (2008) and Tour and He (2008) announced that a memristor of nanometer-size solid-state two-terminal device has been fabricated by a team from the Hewlett-Packard Company. In this memristor, the value (memristance) depends on the magnitude and polarity of the voltage applied to it and the length of the time that the voltage has been applied. When the voltage is turned off, the memristor remembers its most recent value until it is turned on next time. Because of this feature, the passive electronic device has generated unprecedented worldwide interest because of its potential applications (Itoh and Chua 2008; Wang et al. 2010; Kvatinsky et al. 2013). For instance, based on the memristor technique, the next generation computers may be powerful brain-like "neural" computers and turn on instantly without the usual "booting time" currently required in personal computers (Itoh and Chua 2008).

Neural networks can be constructed by nonlinear circuits and have been extensively studied because of their immense potential applications in different areas such as pattern recognition, parallel computing, signal and image processing, and associative memory (Balasubramaniam et al. 2011; Yang et al. 2010; Zhu and Cao 2010; Chen and Song 2010;

Tsukada et al. 2013). Recently, dynamical behaviors of memristor-based neural networks have attracted increasing attention of researchers because this class of neural networks is a new model to emulate the human brain (Itoh and Chua 2009; Thomas 2013). It is known that memristor-based neural networks is a special kind of differential equations with discontinuous right-hand side (Liu and Cao 2011), which indicates that this class of systems may not have any solution in the classical sense. In fact, Filippov proposed a novel method, i.e., Filippov regularization (Filippov 1988), to transform differential equations with discontinuous right-hand side into a differential inclusion (Aubin and Cellina 1984). By utilizing theories of differential inclusion, dynamical behaviors of differential equations with discontinuous right-hand side can be investigated under the framework of Filippov solution (Shen and Cao 2011). Recently, authors in Wu and Zeng (2012, 2013), Zhang et al. (2012, 2013a, b), and Wu et al. (2011, 2012) studied the stability and synchronization of a class of memristor-based neural networks with discrete time delays by using differential inclusion method. However, the main conditions in Wu and Zeng (2012, 2013), Wu et al. (2011, 2012), and Zhang et al. (2012, 2013a, b) were not correct.

The study of synchronization for neural networks with discontinuous right-hand sides is not an easy work, since the conditions for the stability of neural networks with discontinuous right-hand sides cannot be simply utilized to study synchronization. In Liu and Cao (2011), attempted to investigate the synchronization of neural networks with discontinuous activation functions, but the obtained synchronization criterion is local. Moreover, when the usual control techniques are considered, only qusi-synchronization results can be derived (Liu and Yu 2012; Liu et al. 2012). Recently, in Yang and Cao (2013), the authors investigated exponential synchronization of delayed neural networks with discontinuous activations by designing discontinuous state feedback controller and adaptive controller; the authors in Yang et al. (2013) studied finite-time synchronization of complex networks with nonidentical discontinuous nodes by designing special discontinuous state feedback controllers. Although memristor-based neural networks are also belong to the nonlinear systems with discontinuous right hand sides, their discontinuities are different from those of neural networks with discontinuous activations, and hence the analytical technology utilized to study the synchronization of neural networks with discontinuous activations may not be applicable to investigate the synchronization of memristor-based neural networks. Therefore, in this paper, we shall study the drive-response synchronization issue of memristor-based neural networks and improve the results in Wu and Zeng (2012, 2013), Wu et al. (2011, 2012), and Zhang et al. (2012, 2013a, b).

As an important neural network model, Cohen–Grossberg neural networks were firstly introduced by Cohen and Grossberg (1983). Cohen–Grossberg neural networks model is one of the most popular and typical neural network models. Some other models, such as Hopfield neural networks, cellular neural networks, and bidirectional associative memory neural networks, are special cases of the model (Kamel and Xia 2009; Mahdavi and Kurths 2013; Yang et al. 2008, 2011). Stability and synchronization of continuous Cohen–Grossberg neural networks with or without discrete and distributed delays were studied in the literature (Zhu and Cao 2010; He and Cao 2008; Song and Wang 2008). Recently, stability of Cohen–Grossberg neural networks with discontinuous activations were considered in Chen and Song (2010) and Lu and Chen (2008). However, to the best of our knowledge, few published paper considered synchronization control of memristor-based Cohen–Grossberg neural networks. Moreover, according to our study, results on stability of neural networks with discontinuous activations and memristors can not be extended to investigate the synchronization of discontinuous chaotic systems due to the special requirements of stability on the connection weight matrices of the neurons. For example, the connection matrices of discontinuous activations must satisfy the Lyapunov Diagonal Stable (LDS) condition (Chen and Song 2010; Cheng et al. 2007; Di Marco et al. 2012; Forti et al. 2006; Lu and Chen 2008; Wu et al. 2010). However, when these special and strict conditions are satisfied, neural networks usually do not exhibit chaotic behaviors. Therefore, the methods applicable to the stability of neural networks with discontinuous activations and memristors can not be directly employed to study the synchronization of memristor-based Cohen–Grossberg neural networks.

Motivated by the above analysis, this paper proposes a memristor-based Cohen–Grossberg neural networks model with time-varying discrete delays and unbounded distributed delays, and then investigates global exponential synchronization of the model. By using the sign function, we design a novel state feedback controller, which is added to the slave system such that its driven states can globally exponentially synchronize with those in the master system. Based on the characteristics of amplification function, behaved function, and derivative theorem for inverse function, we first get an alternative system from the considered memristor-based Cohen–Grossberg neural networks. By investigating the global exponential synchronization of the alternative system, we obtain the corresponding synchronization criteria of the considered model. The convergence rate is explicitly estimated. Moreover, the conditions utilized in this paper are easy to be verified and improve the conditions derived in Wu and Zeng (2012, 2013), Wu et al. (2011, 2012), and Zhang et al. (2012, 2013a, b). Numerical simulations are given to show the effectiveness of the theoretical results.

The rest of this paper is organized as follows. In Sect. 2, model of memristor-based Cohen–Grossberg neural networks with mixed delays is described. Some necessary assumptions, definitions, and lemmas are also given in this section. Exponential synchronization of the considered model under state feedback control are studied in Sect. 3. In Sect. 4, numerical example is given to show the effectiveness of our results. Conclusions are finally reached in Sect. 5.

Model description and some preliminaries

Referring to the relevant models in Wu and Zeng (2012, 2013), Wu et al. (2011, 2012), and Zhang et al. (2012, 2013a, b) for memristor-based recurrent neural networks, in this paper, we consider a memristor-based Cohen–Grossberg neural network model with mixed delays which is described as follows:

$$\begin{aligned}\dot{u}_i(t) = -a_i(u_i(t))\Bigg\{ & b_i(u_i(t)) - \sum_{j=1}^{n}\Big[w_{ij}(u_i(t))\\ & \times f_j(u_j(t)) + c_{ij}(u_i(t))f_j(u_j(t-\tau_{ij}(t)))\\ & + d_{ij}(u_i(t))\int_{-\infty}^{t}K_{ij}(t-s)f_j(u_j(s))\mathrm{d}s\Big] - I_i\Bigg\},\\ & i = 1,2,\ldots,n,\end{aligned} \tag{1}$$

where $u_i(t)$ denotes the state variable of the ith neuron at time t, I_i is the external input to the ith neuron, $a_i(u_i(t))$ and $b_i(u_i(t))$ represent the amplification function and appropriately behaved function at time t, respectively; the time-varying delay $\tau_{ij}(t)$ corresponds to the finite speed of the axonal signal transmission; $K_{ij}(t)$ is a non-negative bounded scalar function defined on $[0,+\infty)$ describing the delay kernel of the unbounded distributed delay; $w_{ij}(u_i(t)), c_{ij}(u_i(t))$ and $d_{ij}(u_i(t))$ are connection weights of the neural network satisfying the following conditions:

$$w_{ij}(u_i(t)) = \begin{cases}\hat{w}_{ij}, & |u_i(t)| < T_i,\\ \check{w}_{ij}, & |u_i(t)| > T_i,\end{cases} \tag{2}$$

$$c_{ij}(u_i(t)) = \begin{cases}\hat{c}_{ij}, & |u_i(t)| < T_i,\\ \check{c}_{ij}, & |u_i(t)| > T_i,\end{cases} \tag{3}$$

$$d_{ij}(u_i(t)) = \begin{cases}\hat{d}_{ij}, & |u_i(t)| < T_i,\\ \check{d}_{ij}, & |u_i(t)| > T_i,\end{cases} \tag{4}$$

where switching jumps $T_i > 0, \hat{w}_{ij}, \check{w}_{ij}, \hat{c}_{ij}, \check{c}_{ij}, \hat{d}_{ij}, \check{d}_{ij},\ i,j = 1,2,\ldots,n$, are constants.

Remark 1 Model (1) includes the memristor-based recurrent neural networks studied in Wu and Zeng (2012, 2013), Wu et al. (2011, 2012), and Zhang et al. (2012, 2013a, b) as a special case since unbounded distributed delays are also considered in Model (1).When $w_{ij}(u_i(t)), c_{ij}(u_i(t))$,and $d_{ij}(u_i(t))$ are deterministic constants and $K_{ij}(t) = 1$ for $t \in [0,\theta]$ (θ is a positive constant) and $K_{ij}(t) = 0$ for $t > \theta$, the model (1) turns out to the systems studied in Gan (2012), Zhu and Cao (2010) and Song and Wang (2008). Therefore, models in this paper are general.

In order to achieve our main results, the following assumptions are needed:

(H_1) The parameters $w_{ij}(u_i(t)), c_{ij}(u_i(t))$, and $d_{ij}(u_i(t))$ satisfy the conditions (3) and (4), and there exist constants $\tau_{ij} > 0$ such that $0 \le \tau_{ij}(t) \le \tau_{ij}, i,j = 1,2,\ldots, n, t \in \mathbb{R}$.
$(H_2) a_i(u)$ is continuous and there exist positive constants $\underline{a}_i$ and $\bar{a}_i$ such that $0 < \underline{a}_i \le a_i(u) \le \bar{a}_i, u \in \mathbb{R}, i = 1,2,\ldots,n$.
(H_3) There exist positive constants l_i such that $b_i'(u) \ge l_i$, where $b_i'(u)$ denotes the derivative of $b_i'(u), u \in \mathbb{R}$ and $b_i(0) = 0, i = 1,2,\ldots,n$.
(H_4) There exist constants L_i such that $|f_i(x) - f_i(y)| \le L_i|x-y|,\ \forall x,y \in \mathbb{R}, x \ne y, i = 1,2,\ldots,n$.
(H_5) There exist constants M_i such that $|f_i(x)| \le M_i$ for bounded $x, \forall x \in \mathbb{R}, i = 1,2,\ldots,n$.
(H_6) The delay kernels $K_{ij} : [0,+\infty) \to [0,+\infty)$ are real-valued non-negative continuous functions and there exist positive numbers β_{ij} such that $\int_0^{+\infty} K_{ij}(s)\mathrm{d}s \le \beta_{\mathrm{ij}}$, $i = 1,2,\ldots,n$.

From (H_2), the antiderivative of $\frac{1}{a_i(u_i)}$ exists. We choose an antiderivative $h_i(u_i)$ of $\frac{1}{a_i(u_i)}$ that satisfies $h_i(0) = 0$. Obviously, $\frac{\mathrm{d}}{\mathrm{d}u_i}h_i(u_i) = \frac{1}{a_i(u_i)}$. By $a_i(u_i) > 0$, we obtain that $h_i(u_i)$ is strictly monotone increasing about u_i. In view of derivative theorem for inverse function, the inverse function $h_i^{-1}(u_i)$ of $h_i(u_i)$ is differentiable and $\frac{\mathrm{d}}{\mathrm{d}u_i}h_i^{-1}(u_i) = a_i(u_i)$. By (H_3), composition function $b_i(t, h_i^{-1}(z))$ is differentiable. Denote $x_i(t) = h_i(u_i(t))$. It is easy to see that $x_i'(t) = \frac{u_i'(t)}{a_i(u_i(t))}$ and $u_i(t) = h_i^{-1}(x_i(t))$. Substituting these equalities into system (1), we get

$$\begin{aligned}\dot{x}_i(t) = & -b_i\big(h_i^{-1}(x_i(t))\big) + \sum_{j=1}^{n}\Big[w_{ij}\big(h_i^{-1}(x_i(t))\big)\\ & \times f_j\Big(h_j^{-1}(x_j(t))\Big) + c_{ij}\big(h_i^{-1}(x_i(t))\big)\\ & \times f_j\Big(h_j^{-1}(x_j(t-\tau_{ij}(t)))\Big)\\ & + d_{ij}\big(h_i^{-1}(x_i(t))\big)\int_{-\infty}^{t}K_{ij}(t-s)\\ & \times f_j\Big(h_j^{-1}(x_j(s))\Big)\mathrm{d}s\Big] + I_i.\end{aligned} \tag{5}$$

From conditions (2), (3), and (4), one can see that system (1) is a differential equation with discontinuous right-hand side. In this case, the solution of (1) in the conventional sense does not exist. However, we can discuss the dynamical behaviors of system (1) by means of Filippov solution. In the following, we first recall the notation of set-valued map, which is needed to define Filippov solution.

Definition 1 Filippov(1960). The Filippov set-valued map of $f(x)$ at $x \in \mathbb{R}^n$ is defined as follows:

$$F(x) = \bigcap_{\delta>0} \bigcap_{\mu(\Omega)=0} \overline{co}[f(B(x,\delta) \setminus \Omega)],$$

where $\overline{co}[E]$ is the closure of the convex hull of the set $E, B(x,\delta) = \{y : \|y - x\| \le \delta\}$, and $\mu(\Omega)$ is the Lebesgue measure of set Ω. For the convenience of later study, we introduce the following notations: $\bar{w}_{ij} = \max\{\hat{w}_{ij}, \check{w}_{ij}\}$, $\underline{w}_{ij} = \min\{\hat{w}_{ij}, \check{w}_{ij}\}, \bar{c}_{ij} = \max\{\hat{c}_{ij}, \check{c}_{ij}\}, \underline{c}_{ij} = \min\{\hat{c}_{ij}, \check{c}_{ij}\}$, $\bar{d}_{ij} = \max\{\hat{d}_{ij}, \check{d}_{ij}\}, \underline{d}_{ij} = \min\{\hat{d}_{ij}, \check{d}_{ij}\}$.

Based on Definition 1 and theory of differential inclusion, it can be obtained from (5) that

$$\begin{aligned}\dot{x}_i(t) \in -b_i(h_i^{-1}(x_i(t))) + \sum_{j=1}^{n} \Big[\overline{co}\big[\underline{w}_{ij}, \bar{w}_{ij}\big] \\ \times f_j\Big(h_j^{-1}(x_j(t))\Big) + \overline{co}[\underline{c}_{ij}, \bar{c}_{ij}] \\ \times f_j\Big(h_j^{-1}(x_j(t-\tau_{ij}(t)))\Big) \\ + \overline{co}\big[\underline{d}_{ij}, \bar{d}_{ij}\big] \int_{-\infty}^{t} K_{ij}(t-s) \\ \times f_j\Big(h_j^{-1}(x_j(s))\Big)\mathrm{d}s\Big] + I_i. \end{aligned} \tag{6}$$

or equivalently, there exist $\gamma_{ij}(t) \in \overline{co}[\underline{w}_{ij}, \bar{w}_{ij}], \delta_{ij}(t) \in \overline{co}[\underline{c}_{ij}, \bar{c}_{ij}], \zeta_{ij}(t) \in \overline{co}[\underline{d}_{ij}, \bar{d}_{ij}]$ such that

$$\begin{aligned}\dot{x}_i(t) = & - b_i\big(h_i^{-1}(x_i(t))\big) + \sum_{j=1}^{n} [\gamma_{ij}(t) \\ & \times f_j\Big(h_j^{-1}(x_j(t))\Big) + \delta_{ij}(t)\, f_j\Big(h_j^{-1}(x_j(t-\tau_{ij}(t)))\Big) \\ & + \zeta_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-s) \\ & \times f_j\Big(h_j^{-1}(x_j(s))\Big)\mathrm{d}s] + I_i. \end{aligned} \tag{7}$$

Let system (1) be the driving system. We construct a controlled response system described by

$$\begin{aligned}\dot{v}_i(t) = & - a_i(v_i(t))\Bigg\{b_i(v_i(t)) - \sum_{j=1}^{n} \Bigg[w_{ij}(v_i(t)) \\ & \times h_j(v_j(t)) + c_{ij}(v_i(t))\, f_j(v_j(t-\tau_{ij}(t))) + d_{ij}(v_i(t)) \\ & \int_{-\infty}^{t} K_{ij}(t-s) g_j(v_j(s))\mathrm{d}s\Bigg] - I_i\Bigg\} + R_i(t), \end{aligned} \tag{8}$$

where the feedback control term $R_i(t)$ is

$$\begin{aligned}R_i(t) = & - p_i(v_i(t) - u_i(t)) \\ & - \eta_i \mathrm{sign}(v_i(t) - u_i(t)), \quad t \ge 0, \end{aligned} \tag{9}$$

where p_i, η_i are the control gains to be determined.

Similar to the analysis of (5), (6), and (7), we have from (8) and (9) that

$$\begin{aligned}\dot{y}_i(t) = & - b_i(h_i^{-1}(y_i(t))) + \sum_{j=1}^{n} \Big[\bar{\gamma}_{ij}(t) f_j(h_j^{-1}(y_j(t))) \\ & + \bar{\delta}_{ij}(t) f_j(h_j^{-1}(y_j(t-\tau_{ij}(t)))) \\ & + \bar{\zeta}_{ij}(t) \int_{-\infty}^{t} K_{ij}(t-s) f_j(h_j^{-1}(y_j(s)))\mathrm{d}s\Big] \\ & + I_i - \frac{p_i}{a_i(h_i^{-1}(y_i(t)))}(h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t))) \\ & - \frac{\eta_i}{a_i(h_i^{-1}(y_i(t)))} \mathrm{sign}(h_i^{-1}(y_i(t)) \\ & - h_i^{-1}(x_i(t))), \end{aligned} \tag{10}$$

where $y_i(t) = h_i(v_i(t)), \bar{\gamma}_{ij}(t) \in \overline{co}[\underline{w}_{ij}, \bar{w}_{ij}], \bar{\delta}_{ij}(t) \in \overline{co}[\underline{c}_{ij}, \bar{c}_{ij}], \bar{\zeta}_{ij}(t) \in \overline{co}[\underline{d}_{ij}, \bar{d}_{ij}]$

The initial values $\varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))^T$ of (1) and $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T$ of (8) are of the following form

$$u_i(s) = \varphi_i(s), \quad v_i(s) = \phi_i(s), \tag{11}$$

where $s \le 0, i = 1, 2, \ldots, n, \varphi_i(s)$ and $\phi_i(s)$ are continuous functions.

Then the initial values $h(\varphi(s)) = (h_1(\varphi_1(s)), h_2(\varphi_2(s)), \ldots, h_n(\varphi_n(s)))^T$ of (7) and $h(\phi(s)) = (h_1(\phi_1(s)), h_2(\phi_2(s)), \ldots, h_n(\phi_n(s)))^T$ of (10) are of the following form

$$x_i(s) = h_i(\varphi_i(s)), y_i(s) = h_i(\phi_i(s)), \quad s \le 0, \tag{12}$$

where $i = 1, 2, \ldots, n$.

Remark 2 The assumptions in this paper are very mild and can be easily verified. Recently, the authors in Wu and Zeng (2012, 2013), Wu et al. (2011, 2012), and Zhang et al. (2012, 2013a, b) investigated the stability and synchronization of neural networks with memristors under the following condition (H_*):

$$\begin{aligned} & \overline{co}[\underline{w}_{ij}, \bar{w}_{ij}] f_j(x_j) - \overline{co}[\underline{w}_{ij}, \bar{w}_{ij}] f_j(y_j) \\ \subseteq\ & \overline{co}[\underline{w}_{ij}, \bar{w}_{ij}](f_j(x_j) - f_j(y_j)), \\ & \overline{co}[\underline{\delta}_{ij}, \bar{\delta}_{ij}] f_j(x_j) - \overline{co}[\underline{\delta}_{ij}, \bar{\delta}_{ij}] f_j(y_j) \\ \subseteq\ & \overline{co}[\underline{\delta}_{ij}, \bar{w}_{ij}](f_j(x_j) - f_j(y_j)), \\ & \overline{co}[\underline{\zeta}_{ij}, \bar{\zeta}_{ij}] f_j(x_j) - \overline{co}[\underline{\zeta}_{ij}, \bar{\zeta}_{ij}] f_j(y_j) \\ \subseteq\ & \overline{co}[\underline{\zeta}_{ij}, \bar{w}_{ij}](f_j(x_j) - f_j(y_j)). \end{aligned}$$

However, the condition (H_*) is not right. We take the first inclusion relation as example. When $\xi_1 = \underline{w}_{ij}$ and $\xi_2 = \bar{w}_{ij}$, then, for any $x_j, y_j \in \mathbb{R}$, there exists $\xi \in \overline{co}[\underline{w}_{ij}, \bar{w}_{ij}]$ such that

$$\begin{aligned} \xi_1 f_j(x_j) - \xi_2 f_j(y_j) &= \underline{w}_{ij} f_j(x_j) - \bar{w}_{ij} f_j(y_j) \\ &= \xi(f_j(x_j) - f_j(y_j)). \end{aligned} \tag{13}$$

Obviously, $\xi \neq \underline{w}_{ij}$ and $\xi \neq \bar{w}_{ij}$. Otherwise, $\xi = \underline{w}_{ij} = \bar{w}_{ij}$, which contradicts the condition $\underline{w}_{ij} < \bar{w}_{ij}$. Hence,

$$\underline{w}_{ij} < \xi < \bar{w}_{ij}. \tag{14}$$

It follows from (13) and (14) that

$$f_j(x_j) = \frac{\bar{w}_{ij} - \xi}{\underline{w}_{ij} - \xi} f_j(y_j), \tag{15}$$

which means that

$$f_j(x_j) - f_j(y_j) = \left[\frac{\bar{w}_{ij} - \xi}{\underline{w}_{ij} - \xi} - 1\right] f_j(y_j). \tag{16}$$

On the other hand, one has from (14) that $\frac{\bar{w}_{ij}-\xi}{\underline{w}_{ij}-\xi} < 0$. So, the equality (16) implies that $f_j(x_j) < 0$. From the arbitrariness of $x_j \in \mathbb{R}$ we get that $f_j(u)$ is an negative-valued function on $\mathbb{R}$. However, considering $f_j(x_j) < 0$ and $\frac{\bar{w}_{ij}-\xi}{\underline{w}_{ij}-\xi} < 0$, we obtain from (15) that $f_j(y_j) > 0$, which implies that $f_j(u)$ is a positive-valued function on $\mathbb{R}$, this is a contradiction. Hence, there is no $\xi \in \overline{co}[\underline{w}_{ij}, \bar{w}_{ij}]$ such that the equality (13) holds, and so the first inclusion relation (H_*) is not correct.

If $f_j(u), j = 1, 2, \ldots, n, u \in \mathbb{R}$ satisfy the Lipschitz condition, i.e., there exist positive constants L_j such that $|f_j(x_j) - f_j(y_j)| \leq L_j |x_j - y_j|$ for all $x_j, y_j \in \mathbb{R}$, then other contradiction can also derived. It is obtained from (16) that

$$\begin{aligned} |f_j(x_j) - f_j(y_j)| &= \left| \left(\frac{\bar{w}_{ij} - \xi}{\underline{w}_{ij} - \xi} - 1 \right) f_j(y_j) \right| \\ &\leq L_j |x_j - y_j|. \end{aligned} \tag{17}$$

Letting $x_j \to y_j$, the above inequality implies that $f_j(y_j) = 0$, which contradicts the practical meaning of neural networks. This contradiction also means the Lipschitz condition of the activation and the condition (H_*) cannot coexist.

Definition 2 The controlled system (8) is said to be globally exponentially synchronized with system (1) if there exist positive constants M and α such that

$$|v_i(t) - u_i(t)| \leq M \|\varphi(s) - \phi(s)\| \exp(-\alpha t),$$

$i = 1, 2, \ldots, n$, *hold for* $t \geq 0$, *where* $\|\varphi(s) - \phi(s)\| = \sup_{s \leq 0} \max_{1 \leq i \leq n} |\varphi_i(s) - \phi_i(s)|$.

Lemma 1 *(Chain rule) Clarke (1987). If $V(x) : \mathbb{R}^n \to \mathbb{R}$ is C-regular and $x(t)$ is absolutely continuous on any compact subinterval of $[0, +\infty)$, then $x(t)$ and $V(x(t)) : [0, +\infty) \to \mathbb{R}$ are differentiable for a.a. $t \in [0, +\infty)$ and*

$$\frac{\mathrm{d}}{\mathrm{d}t} V(x(t)) = \gamma(t) \dot{x}(t), \quad \forall \gamma(t) \in \partial V(x(t)), \tag{18}$$

where $\partial V(x(t))$ is the Clark generalized gradient of V at $x(t)$.

Synchronization control of memristor-based Cohen–Grossberg neural networks

In this section, synchronization criteria for memristor-based Cohen–Grossberg neural networks with time-varying and unbounded distributed delays under the controller (9) is derived by rigorous mathematical proof. One corollary, which is applicable to memristor-based recurrent networks, is also derived.

Theorem 1 *Assume (H_1)–(H_6) hold and the following inequalities are satisfied*:

$$\begin{aligned} p_i > \frac{\bar{a}_i}{\underline{a}_i} \Bigg(&-\underline{a}_i l_i + \sum_{j=1}^{n} \bar{a}_j L_j (\bar{w}_{ij} + \bar{c}_{ij} \\ &+ \bar{d}_{ij} \beta_{ij}) \Bigg) = \Xi_i, \end{aligned} \tag{19}$$

$$\begin{aligned} \eta_i \geq \bar{a}_i \Bigg(&\sum_{j=1}^{n} [|\hat{w}_{ij} - \check{w}_{ij}| + |\hat{c}_{ij} - \check{c}_{ij}| \\ &+ |\hat{d}_{ij} - \check{d}_{ij}| \beta_{ij}] M_j \Bigg) = \Lambda_i, \end{aligned} \tag{20}$$

$i = 1, 2, \ldots, n$, then the controlled system (8) is globally exponentially synchronized with system (1) under the controller (9).

Proof Set $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T = y(t) - x(t)$. It follows from systems (7) and (10) that

$$\begin{aligned}\dot{z}_i(t) =& -[b_i(h_i^{-1}(y_i(t))) - b_i(h_i^{-1}(x_i(t)))]\\&+\sum_{j=1}^{n}\left[\bar{\gamma}_{ij}(t)f_j(h_j^{-1}(y_j(t))). - \gamma_{ij}(t)f_j(h_j^{-1}(x_j(t)))\right]\\&+\sum_{j=1}^{n}\left[\bar{\delta}_{ij}(t)f_j(h_j^{-1}(y_j(t-\tau_{ij}(t))))\right.\\&\left.-\delta_{ij}(t)f_j(h_j^{-1}(x_j(t-\tau_{ij}(t))))\right]\\&+\sum_{j=1}^{n}\left[\bar{\zeta}_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)f_j(h_j^{-1}(y_j(s)))\mathrm{d}s.\right.\\&\left.-\zeta_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)f_j(h_j^{-1}(x_j(s)))\mathrm{d}s\right]\\&-\frac{p_i}{a_i(h_i^{-1}(y_i(t)))}(h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t)))\\&-\frac{\eta_i}{a_i(h_i^{-1}(y_i(t)))}\mathrm{sign}(h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t))).\end{aligned} \tag{21}$$

Construct the function $q_i(\lambda)$ as follows:

$$\begin{aligned}q_i(\lambda) =& \lambda - \underline{a}_i l_i - \frac{p_i \underline{a}_i}{\bar{a}_i} + \sum_{j=1}^{n}\bar{a}_j L_j\Big(\bar{w}_{ij} + \bar{c}_{ij}e^{\lambda\tau_{ij}}\\&+\bar{d}_{ij}\int_0^{+\infty}|k_{ij}(s)|e^{\lambda s}\mathrm{d}s\Big), \quad i = 1,2,\ldots,n.\end{aligned} \tag{22}$$

It follows from (19) that $q_i(0) < 0$. Moreover, $q_i(\lambda), i = 1,2,\ldots,n$, are continuous functions about $\lambda \in \mathbb{R}$, and

$$q_i'(\lambda) = 1 + \sum_{j=1}^{n}\bar{a}_j L_j(\tau_{ij}\bar{c}_{ij}e^{\lambda\tau_{ij}} + \bar{d}_{ij}\int_0^{+\infty}|k_{ij}(s)|se^{\lambda s}\mathrm{d}s) > 0,$$

and $q_i(+\infty) = +\infty$, hence $q_i(\lambda), i = 1,2,\ldots,n$, are strictly monotonically increasing functions. Therefore, for any $i \in \{1,2,\ldots,n\}$, there is a unique $\lambda_i > 0$ such that

$$\begin{aligned}&\lambda_i - \underline{a}_i l_i - \frac{p_i \underline{a}_i}{\bar{a}_i} + \sum_{j=1}^{n}\bar{a}_j L_j(\bar{w}_{ij} + \bar{c}_{ij}e^{\lambda_i\tau_{ij}}\\&+\bar{d}_{ij}\int_0^{+\infty}|k_{ij}(s)|e^{\lambda_i s}\mathrm{d}s) = 0.\end{aligned}$$

Taking $\alpha = \min\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ yields

$$\begin{aligned}q_i(\alpha) =& \alpha - \underline{a}_i l_i - \frac{p_i \underline{a}_i}{\bar{a}_i} + \sum_{j=1}^{n}\bar{a}_j L_j(\bar{w}_{ij} + \bar{c}_{ij}e^{\alpha\tau_{ij}}\\&+\bar{d}_{ij}\int_0^{+\infty}|k_{ij}(s)|e^{\alpha s}\mathrm{d}s) \le 0, \quad i = 1,2,\ldots,n.\end{aligned} \tag{23}$$

Define a Lyapunov functional by $V_i = e^{\alpha t}|z_i(t)|, i = 1,2,\ldots,n$. In view of Lemma 1, it can be obtained from system (21) that

$$\begin{aligned}\dot{V}_i(t) =& e^{\alpha t}\mathrm{sign}(z_i(t))\{-[b_i(h_i^{-1}(y_i(t)))\\&-b_i(h_i^{-1}(x_i(t)))] + \sum_{j=1}^{n}\left[\bar{\gamma}_{ij}(t)f_j(h_j^{-1}(y_j(t)))\right.\\&\left.-\gamma_{ij}(t)f_j(h_j^{-1}(x_j(t)))\right]\\&+\sum_{j=1}^{n}\left[\bar{\delta}_{ij}(t)f_j(h_j^{-1}(y_j(t-\tau_{ij}(t))))\right.\\&\left.-\delta_{ij}(t)f_j(h_j^{-1}(x_j(t-\tau_{ij}(t))))\right]\\&+\sum_{j=1}^{n}\left[\bar{\zeta}_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)f_j(h_j^{-1}(y_j(s)))\mathrm{d}s\right.\\&\left.-\zeta_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)f_j(h_j^{-1}(x_j(s)))\mathrm{d}s\right]\\&-\frac{p_i}{a_i(h_i^{-1}(y_i(t)))}(h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t)))\\&-\frac{\eta_i}{a_i(h_i^{-1}(y_i(t)))}\mathrm{sign}(h_i^{-1}(y_i(t))\\&-h_i^{-1}(x_i(t)))\} + \alpha e^{\alpha t}|z_i(t)|.\end{aligned} \tag{24}$$

Since $b_i(u)$ and $h_i^{-1}(\lambda)$ are strictly monotonically increasing and differentiable, $b_i(0) = 0$, and $h_i^{-1}(0) = 0, b_i(h_i^{-1}(\lambda))$ is strictly monotonically increasing and differentiable about $\lambda \in \mathbb{R}$ and

$$\begin{aligned}&b_i(h_i^{-1}(y_i(t))) - b_i(h_i^{-1}(x_i(t)))\\&= b_i'(h_i^{-1}(\lambda))|_{\lambda=\xi}(y_i(t) - x_i(t)),\end{aligned} \tag{25}$$

where $b_i'(h_i^{-1}(\lambda))|_{\lambda=\xi}$ denote the derivative of $b_i(h_i^{-1}(\lambda))$ at the point $\lambda = \xi, \xi$ is between $y_i(t) and x_i(t)$. It is obvious that $b_i'(h_i^{-1}(\lambda))|_{\lambda=\xi}$ is unique for any $y_i(t) and x_i(t)$. Moreover, $b_i'(h_i^{-1}(\lambda))|_{\lambda=\xi} \ge \underline{a}_i l_i$. Therefore, it is obtained from (25) that

$$\begin{aligned}&-\mathrm{sign}(z_i(t))b_i(h_i^{-1}(y_i(t))) - b_i(h_i^{-1}(x_i(t)))\\&= -b_i'(h_i^{-1}(\lambda))|_{\lambda=\xi}|z_i(t)| \le -\underline{a}_i l_i|z_i(t)|,\end{aligned} \tag{26}$$

and

$$\begin{aligned}&-\mathrm{sign}(z_i(t))\frac{p_i}{a_i(h_i^{-1}(y_i(t)))}(h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t)))\\&\le -\frac{p_i \underline{a}_i}{\bar{a}_i}|z_i(t)|.\end{aligned} \tag{27}$$

Moreover, since $h_i^{-1}(\lambda)$ strictly monotone increasing and $h_i^{-1}(0) = 0$, we get $\mathrm{sign}(h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t))) = \mathrm{sign}(z_i(t))$. Thus,

$$- \operatorname{sign}(z_i(t))\frac{\eta_i}{a_i(h_i^{-1}(y_i(t)))}\operatorname{sign}(h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t))) = -\frac{\eta_i}{a_i(h_i^{-1}(y_i(t)))} \le -\frac{\eta_i}{\bar{a}_i}. \tag{28}$$

It is derived from $(H_1), (H_4), (H_5)$ and (H_6) that

$$\begin{aligned}&|\bar{\gamma}_{ij}(t)f_j(h_j^{-1}(y_j(t))) - \gamma_{ij}(t)f_j(h_j^{-1}(x_j(t)))|\\ &\le |\bar{\gamma}_{ij}(t)f_j(h_j^{-1}(y_j(t))) - \bar{\gamma}_{ij}(t)f_j(h_j^{-1}(x_j(t)))|\\ &+ |\bar{\gamma}_{ij}(t)f_j(h_j^{-1}(x_j(t))) - \gamma_{ij}(t)f_j(h_j^{-1}(x_j(t)))|\\ &\le \bar{w}_{ij}L_j\bar{a}_j|z_j(t)| + |\hat{w}_{ij} - \check{w}_{ij}|M_j,\end{aligned} \tag{29}$$

$$\begin{aligned}&|\bar{\delta}_{ij}(t)f_j(h_j^{-1}(y_j(t-\tau_{ij}(t))))\\ &- \delta_{ij}(t)f_j(h_j^{-1}(x_j(t-\tau_{ij}(t))))|\\ &\le \bar{c}_{ij}L_j\bar{a}_j|z_j(t-\tau_{ij}(t))| + |\hat{c}_{ij} - \check{c}_{ij}|M_j,\end{aligned} \tag{30}$$

and

$$\begin{aligned}&\left|\bar{\zeta}_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)f_j(h_j^{-1}(y_j(s)))\mathrm{d}s\right.\\ &\left.-\zeta_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-s)f_j(h_j^{-1}(x_j(s)))\mathrm{d}s\right|\\ &\le \bar{d}_{ij}L_j\bar{a}_j\int_{-\infty}^{t} K_{ij}(t-s)|z_j(s)|\mathrm{d}s + |\hat{d}_{ij} - \check{d}_{ij}|\beta_{ij}M_j.\end{aligned} \tag{31}$$

Substituting (26)–(9) into (24) produces the following inequality:

$$\begin{aligned}\dot{V}_i(t) \le e^{\alpha t}\Bigg\{&\left(\alpha - \underline{a}_i l_i - \frac{p_i\underline{a}_i}{\bar{a}_i}\right)|z_i(t)|\\ &+ \sum_{j=1}^{n}\Big[\bar{w}_{ij}L_j\bar{a}_j|z_j(t)| + \bar{c}_{ij}L_j\bar{a}_j|z_j(t-\tau_{ij}(t))|\\ &+\bar{d}_{ij}L_j\bar{a}_j\int_{-\infty}^{t} K_{ij}(t-s)|z_j(s)|\mathrm{d}s\Big]\\ &+ \sum_{j=1}^{n}[|\hat{w}_{ij} - \check{w}_{ij}| + |\hat{c}_{ij} - \check{c}_{ij}|\\ &+|\hat{d}_{ij} - \check{d}_{ij}|\beta_{ij}]M_j - \frac{\eta_i}{\bar{a}_i}\Bigg\}.\end{aligned} \tag{32}$$

Considering the condition (20), we get from (32) that

$$\begin{aligned}\dot{V}_i(t) \le &\left(\alpha - \underline{a}_i l_i - \frac{p_i\underline{a}_i}{\bar{a}_i}\right)V_i(t) + \sum_{j=1}^{n}[\bar{w}_{ij}L_j\bar{a}_jV_j(t)\\ &+ \bar{c}_{ij}e^{\alpha\tau_{ij}}L_j\bar{a}_jV_j(t-\tau_{ij}(t)) + \bar{d}_{ij}L_j\bar{a}_j\\ &\int_{-\infty}^{t} K_{ij}(t-s)e^{\alpha(t-s)}V_j(s)\mathrm{d}s], \quad i = 1,2,\ldots,n.\end{aligned} \tag{33}$$

It is obvious that

$$|z_i(t)| \le \|h(\varphi) - h(\phi)\| \le \|h(\varphi) - h(\phi)\|e^{-\alpha t}$$

for $t \le 0, i = 1,2,\ldots,n$. We claim that

$$V_i(t) = |z_i(t)|e^{\alpha t} \le \|h(\varphi) - h(\phi)\| \tag{34}$$

for all $t \ge 0, i = 1,2,\ldots,n$. Contrarily, there must exists $i_0 \in \{1,2,\ldots,n\}$ and $\tilde{t} > 0$ such that

$$V_{i_0}(\tilde{t}) = \|h(\varphi) - h(\phi)\|, \quad \dot{V}_{i_0}(\tilde{t}) > 0, \tag{35}$$

and

$$V_i(t) \le \|h(\varphi) - h(\phi)\|, \quad \forall t \le \tilde{t}, i = 1,2,\ldots,n. \tag{36}$$

Together with (33), (35), and (36), we obtain that

$$\begin{aligned}0 &< \dot{V}_{i_0}(\tilde{t})\\ &\le \left(\alpha - \underline{a}_{i_0}l_{i_0} - \frac{p_{i_0}\underline{a}_{i_0}}{\bar{a}_{i_0}}\right)V_{i_0}(\tilde{t})\\ &\quad + \sum_{j=1}^{n}[\bar{w}_{i_0j}L_j\bar{a}_jV_j(t) + \bar{c}_{i_0j}e^{\alpha\tau_{i_0j}}L_j\bar{a}_j\times\\ &\quad V_j(\tilde{t} - \tau_{i_0j}(\tilde{t}))\\ &\quad +\bar{d}_{i_0j}L_j\bar{a}_j\int_{-\infty}^{\tilde{t}} K_{i_0j}(\tilde{t}-s)e^{\alpha(\tilde{t}-s)}V_j(s)\mathrm{d}s]\\ &\le \|h(\varphi) - h(\phi)\|\{\alpha - \underline{a}_{i_0}l_{i_0} - \frac{p_{i_0}\underline{a}_{i_0}}{\bar{a}_{i_0}}\\ &\quad + \sum_{j=1}^{n}\bar{a}_jL_j(\bar{w}_{i_0j} + \bar{c}_{i_0j}e^{\alpha\tau_{i_0j}}\\ &\quad +\bar{d}_{i_0j}\int_{0}^{+\infty}|k_{i_0j}(s)|e^{\alpha s}\mathrm{d}s)\}.\end{aligned}$$

Hence,

$$\begin{aligned}0 < \alpha - \underline{a}_{i_0}l_{i_0} - \frac{p_{i_0}\underline{a}_{i_0}}{\bar{a}_{i_0}} + \sum_{j=1}^{n}\bar{a}_jL_j(\bar{w}_{i_0j} + \bar{c}_{i_0j}e^{\alpha\tau_{i_0j}}\\ +\bar{d}_{i_0j}\int_{0}^{+\infty}|k_{i_0j}(s)|e^{\alpha s}\mathrm{d}s),\end{aligned}$$

which contradicts (23). Hence (34) holds. It follows that

$$|z_i(t)| \le \|h(\varphi) - h(\phi)\|e^{-\alpha t}, \tag{37}$$

for $\forall t \ge 0, i = 1,2,\ldots,n$. It follows from (37) that

$$\begin{aligned}|v_i(t) - u_i(t)| &= |h_i^{-1}(y_i(t)) - h_i^{-1}(x_i(t))|\\ &\le \bar{a}_i|z_i(t)|\\ &\le \frac{\bar{a}}{\underline{a}}\|\varphi - \phi\|e^{-\alpha t}\end{aligned} \tag{38}$$

for $\forall t \geq 0, i = 1, 2, \ldots, n,$ where $\bar{a} = \max\{\bar{a}_i, i = 1, 2, \ldots, n\}$ and $\underline{a} = \min\{\underline{a}_i, i = 1, 2, \ldots, n\}$. According to Definition 2, the controlled system (8) is globally exponentially synchronized with system (1) under the controller (9). This completes the proof.

Remark 3 We first transform the Cohen–Grossberg neural network (1) and (8) into (7) and (10) respectively, then we get the exponential synchronization criteria between (1) and (8) through investigating the exponential synchronization criteria between (7) and (10). By this way, we need not to introduce special condition as those in Gan (2012) for Cohen–Grossberg neural network to derive synchronization criteria. Moreover, the transformation technique used in this paper also enables us to simplify Lyapunov function to prove our main results. However, the conventional Lyapunov functions for studying stability and synchronization of Cohen–Grossberg neural network are not so simple.

Remark 4 How to deal with the general amplification function $a_i(u_i(t))$ is a key technology in studying synchronization of Cohen–Grossberg neural networks. In Zhu and Cao (2010) and He and Cao (2008), the amplification function $a_i(u_i(t))$ was constant, which simplified the research greatly. Hence the methods in Zhu and Cao (2010) and He and Cao (2008) are invalid for the models of this paper.

When $\underline{a}_i = \bar{a}_i = 1$ in (H_2), then the memristor-based Cohen–Grossberg neural networks in this paper turn out to the models considered in Wu and Zeng (2012, 2013), Wu et al. (2011, 2012), and Zhang et al. (2012, 2013a, b). We derive the following corollary from Theorem 1.

Corollary 1 *Assume (H_1)–(H_6) hold and $\underline{a}_i = \bar{a}_i = 1$, and the following inequalities are satisfied:*

$$p_i > -l_i + \sum_{j=1}^{n} L_j(\bar{w}_{ij} + \bar{c}_{ij} + \bar{d}_{ij}\beta_{ij}), \tag{39}$$

$$\eta_i \geq \sum_{j=1}^{n} [|\hat{w}_{ij} - \check{w}_{ij}| + |\hat{c}_{ij} - \check{c}_{ij}| + |\hat{d}_{ij} - \check{d}_{ij}|\beta_{ij}]M_j, \tag{40}$$

$i = 1, 2, \ldots, n$, then the controlled system (8) is globally exponentially synchronized with system (1) under the controller (9).

Remark 5 The designed controller (9) synchronizes the memristor-based Cohen–Grossberg neural networks effectively. One may notice that the designed controller consists two parts: $-p_i(v_i(t) - u_i(t))$ and $-\eta_i \mathrm{sign}(v_i(t) - u_i(t))$. *It can be seen from the proof of Theorem 1 that the part $-\eta_i \mathrm{sign}(v_i(t) - u_i(t))$ in the controller plays an important role in dealing with the uncertain differences between the Filippov solutions of the drive and response systems, while the other part $-p_i(v_i(t) - u_i(t))$ is to drive the state of the slave system to synchronize with the master system.*

Examples and simulations

In this section, numerical example is given to show the effectiveness of our theoretical results obtained above.

Consider a memristor-based Cohen–Grossberg neural network model with mixed delays as follows:

$$\begin{aligned}\dot{u}_i(t) = &- a_i(u_i(t))\Bigg\{b_i(u_i(t)) - \sum_{j=1}^{2}\Big[w_{ij}(u_i(t)) \\ &\times f_j(u_j(t)) + c_{ij}(u_i(t))f_j(u_j(t - \tau_{ij}(t))) \\ &+ d_{ij}(u_i(t))\int_{-\infty}^{t} K_{ij}(t-s)f_j(u_j(s))\mathrm{d}s\Big] \\ &- I_i\Bigg\}, i = 1, 2,\end{aligned} \tag{41}$$

where $a_1(u_1) = 6 + \frac{1}{1+u_1^2}, a_2(u_2) = 3 - \frac{1}{1+u_2^2}, b_1(u_1) = 1.61 u_1 + \sin(u_1), b_2(u_2) = 1.45u_2 + \sin(u_2), I_1 = -0.01, I_1 = -0.12, \tau_{11}(t) = 1 - 0.2|\sin(t)|, \tau_{12}(t) = 0.9 - 0.1|\cos(t)|, \tau_{21}(t) = |\sin(t)|, \tau_{22}(t) = |\cos(t)|, K_{ij}(t) = e^{-0.5t}, f_i(u_i) = \tanh(u_i),\ i, j = 1, 2,$

$$w_{11}(u_1) = \begin{cases} 1.81, & |u_1| < 0.3, \\ 2.2, & \|u_1| > 0.3, \end{cases}$$

$$w_{12}(u_1) = \begin{cases} -0.14, & \|u_1| < 0.3, \\ 0.12, & \|u_1| > 0.3, \end{cases}$$

$$w_{21}(u_2) = \begin{cases} -1.9, & \|u_2| < 1, \\ -2.2, & \|u_2| > 1, \end{cases}$$

$$w_{22}(u_2) = \begin{cases} 5, & \|u_2| < 1, \\ 5.2, & \|u_2| > 1, \end{cases}$$

$$c_{11}(u_1) = \begin{cases} -0.95, & \|u_1| < 0.3, \\ -1.3, & \|u_1| > 0.3, \end{cases}$$

$$c_{12}(u_1) = \begin{cases} 0.08, & \|u_1| < 0.3, \\ 0.15, & \|u_1| > 0.3, \end{cases}$$

$$c_{21}(u_2) = \begin{cases} -0.2, & \|u_2| < 1, \\ -0.18, & \|u_2| > 1, \end{cases}$$

$$c_{22}(u_2) = \begin{cases} -2.5, & \|u_2| < 1, \\ -2.3, & \|u_2| > 1, \end{cases}$$

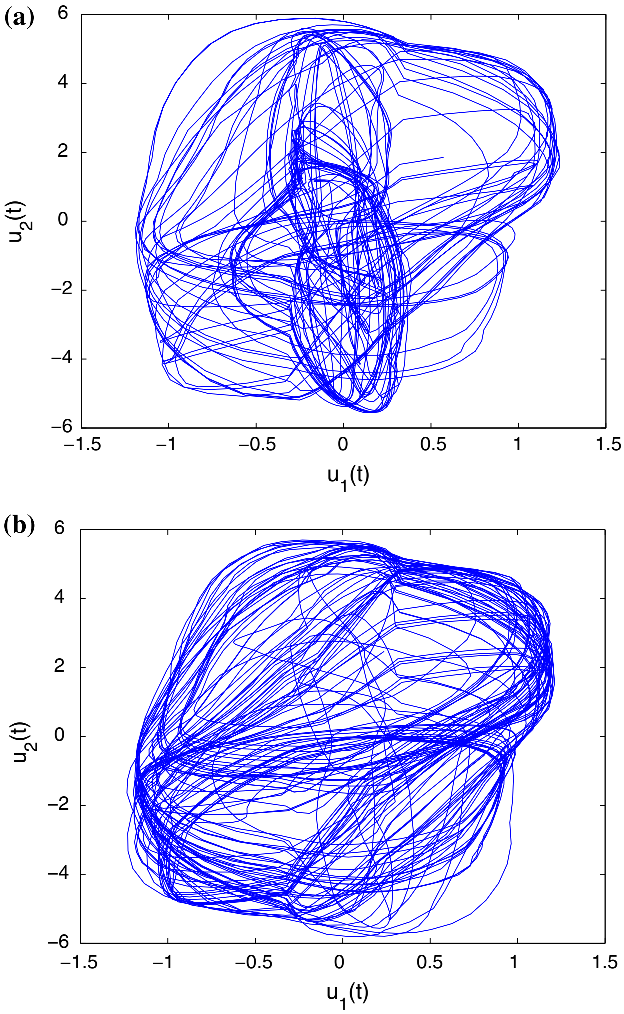

Fig. 1 Trajectories of system (41) with different initial values: **a** $u(t) = (-0.2, 1.2)^T, t \in [-5, 0]$, $u(t) = 0$ for $t \in (-\infty, -5)$; **b** $u(t) = (0.4, 0.6)^T, t \in [-5, 0]$, $u(t) = 0$ for $t \in (-\infty, -5)$.

$$d_{11}(u_1) = \begin{cases} 0.6, \|u_1\| < 0.3, \\ 0.65, \|u_1\| > 0.3, \end{cases}$$
$$d_{12}(u_1) = \begin{cases} 0.12, \|u_1\| < 0.3, \\ -0.12, \|u_1\| > 0.3, \end{cases}$$
$$d_{21}(u_2) = \begin{cases} -0.2, \|u_2\| < 1, \\ -0.18, \|u_2\| > 1, \end{cases}$$
$$d_{22}(u_2) = \begin{cases} -0.1, \|u_2\| < 1, \\ -0.12, \|u_2\| > 1. \end{cases}$$

Obviously, the assumptions $(H_1) - (H_6)$ are satisfied with $\tau_{11} = 1.2, \tau_{12} = 1, \tau_{21} = \tau_{22} = 1, \underline{a}_1 = 6, \bar{a}_1 = 7, \underline{a}_2 = 2$, $\bar{a}_2 = 3, l_1 = 0.61, l_2 = 0.45, L_i = 1$, and $\beta_{ij} = 2, i, j = 1, 2$. Figure 1 describes trajectories of (41) with different initial valves. Moreover, when $u(t) = (-0.2, 1.2)^T, t \in [-5, 0], u(t) = 0$ for $t \in (-\infty, -5)$, we have $M_1 = 0.6206 and M_2 = 1$.

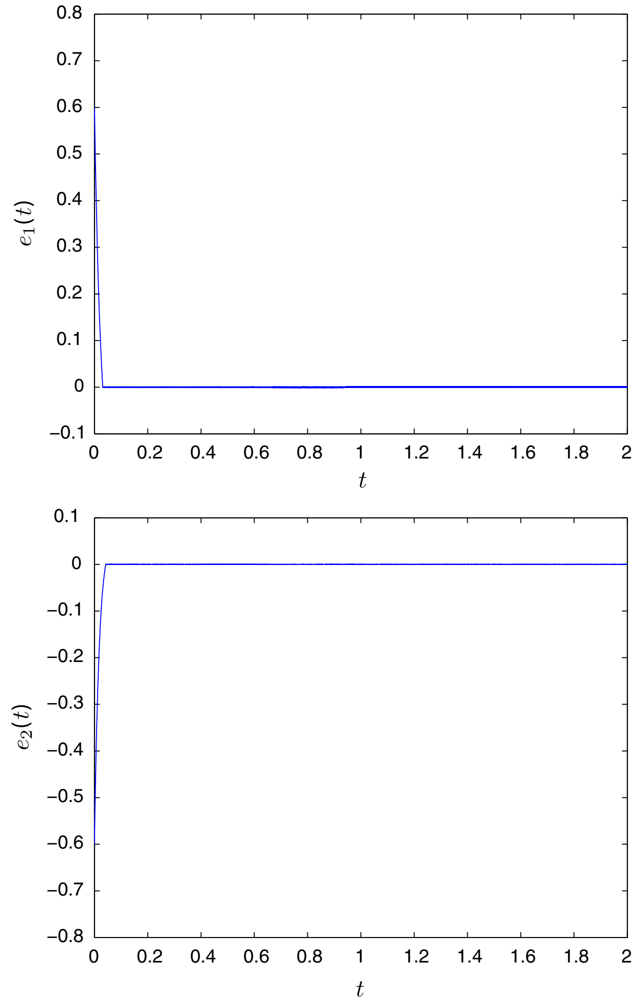

Fig. 2 Time responses synchronization errors $e_1(t)$ (*upper*) and $e_2(t)$ (*lower*) between (41) and (42) under the controller (9)

The controlled response system is described by

$$\begin{aligned} \dot{v}_i(t) = & -a_i(v_i(t))\Bigg\{b_i(v_i(t)) - \sum_{j=1}^{2}\Bigg[w_{ij}(v_i(t)).. \\ & \times h_j(v_j(t)) + c_{ij}(v_i(t))f_j(v_j(t - \tau_{ij}(t))) \\ & + d_{ij}(v_i(t)) \int_{-\infty}^{t} K_{ij}(t-s)g_j(v_j(s))\mathrm{d}s\Bigg] - I_i\Bigg\} \\ & + R_i(t), \quad i = 1, 2, \end{aligned} \tag{42}$$

where the feedback control $R_i(t)$ is defined in ((9).

By simple computation, we have $\Xi_1 = 36.8550$, $\Xi_1 = 63.78$, $\Lambda_1 = 9.0763$, and $\Lambda_2 = 1.9456$. According to Theorem 1, the response system (42) can globally exponentially synchronize with the drive system (41) if we take $p_1 = 67, p_2 = 64, \eta_1 = \Lambda_1$, and $\eta_2 = \Lambda_2$. In the simulations, the initial condition of system (41) and (42) are the same as those of (a) and (b) in the Fig. 1, respectively. Figure 2 shows the time responses of synchronization errors, which implies that the states of the two systems realize synchronization quickly as time goes.

Remark 6 Figure 2 shows that initial values have important effects on the trajectories of memristor-based neural networks. This is because the parameters of memristor-based neural networks depend on the states. Moreover, the numerical example demonstrates that the designed controller (9) is powerful to synchronize memristor-based neural networks even though the different trajectories of the coupled memristor-based neural networks.

Conclusions

In this paper, global exponential synchronization of memristor-based Cohen–Grossberg neural networks model with time-varying discrete delays and unbounded distributed delays has been studied. The considered model is general and covers most of the existing neural network models. By adding a new controller to the response system, this paper shows theoretically and numerically that the response system can globally exponentially synchronize with the drive system, where the synchronization criteria are easily verified. As a by product, numerical simulations also show that the initial values of the memristor-based Cohen–Grossberg neural networks have key effects on their trajectories.

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References

Aubin J, Cellina A (1984) Differential inclusions. Springer, Berlin

Balasubramaniam P, Chandran R, Theesar SJS (2011) Synchronization of chaotic nonlinear continuous neural networks with time-varying delay. Cogn Neurodyn 5(4):361–371

Chen X, Song Q (2010) Global exponential stability of the periodic solution of delayed cohen–grossberg neural networks with discontinuous activations. Neurocomputing 73(16):3097–3104

Cheng CY, Lin KH, Shih CW (2007) Multistability and convergence in delayed neural networks. Phys D 225(1):61–74

Chua L (1971) Memristor-the missing circuit element. IEEE Trans Circuit Theory 18(5):507–519

Chua LO, Kang SM (1976) Memristive devices and systems. Proc IEEE 64(2):209–223

Clarke F (1987) Optimization and nonsmooth analysis. 5 Society for Industrial Mathematics

Cohen MA, Grossberg S (1983) Absolute stability of global pattern formation and parallel memory storage by competitive neural networks. IEEE Trans Syst Man Cybern 13(5):815–826

Di Marco M, Forti M, Grazzini M, Pancioni L (2012) Limit set dichotomy and multistability for a class of cooperative neural networks with delays. IEEE Trans Neural Netw Learn Syst 23(9):1473–1485

Filippov A (1960) Differential equations with discontinuous right-hand side. Matematicheskii Sbornik 93(1):99–128

Filippov A (1988) Differential equations with discontinuous right-hand sides. Kluwer, Dordrecht

Forti M, Grazzini M, Nistri P, Pancioni L (2006) Generalized lyapunov approach for convergence of neural networks with discontinuous or non-lipschitz activations. Phys D 214(1):88–99

Gan Q (2012) Adaptive synchronization of cohen–grossberg neural networks with unknown parameters and mixed time-varying delays. Commun Nonlinear Sci Numer Simul 17(7):3040–3049

He W, Cao J (2008) Adaptive synchronization of a class of chaotic neural networks with known or unknown parameters. Phys Lett A 372(4):408–416

Itoh M, Chua LO (2008) Memristor oscillators. Int J Bifurcation Chaos 18(11):3183–3206

Itoh M, Chua LO (2009) Memristor cellular automata and memristor discrete-time cellular neural networks. Int J Bifurcation Chaos 19(11):3605–3656

Kamel MS, Xia Y (2009) Cooperative recurrent modular neural networks for constrained optimization: a survey of models and applications. Cogn Neurodyn 3(1):47–81

Kvatinsky S, Friedman EG, Kolodny A, Weiser UC (2013) Team: threshold adaptive memristor model. IEEE Trans Circuits Syst 60(1):211–221

Liu X, Cao J (2011) Local synchronization of one-to-one coupled neural networks with discontinuous activations. Cogn Neurodyn 5(1):13–20

Liu X, Yu W (2012) Quasi-synchronization of delayed coupled networks with non-identical discontinuous nodes. Adv Neural Netw–ISNN 7367:274–284

Liu X, Cao J, Yu W (2012) Filippov systems and quasi-synchronization control for switched networks. Chaos 22(3):033110–033110

Lu W, Chen T (2008) Almost periodic dynamics of a class of delayed neural networks with discontinuous activations. Neural Comput 20(4):1065–1090

Mahdavi N, Kurths J (2013) Synchrony based learning rule of hopfield like chaotic neural networks with desirable structure. Cogn Neurodyn. doi:10.1007/s11571-013-9260-2

Shen J, Cao J (2011) Finite-time synchronization of coupled neural networks via discontinuous controllers. Cogn Neurodyn 5(4):373–385

Song Q, Wang Z (2008) Stability analysis of impulsive stochastic cohen–grossberg neural networks with mixed time delays. Phys A 387(13):3314–3326

Strukov DB, Snider GS, Stewart DR, Williams RS (2008) The missing memristor found. Nature 453(7191):80–83

Thomas A (2013) Memristor-based neural networks. J Phys D 46(9):093001

Tour JM, He T (2008) The fourth element. Nature 453(7191):42–43

Tsukada H, Yamaguti Y, Tsuda I (2013) Transitory memory retrieval in a biologically plausible neural network model. Cogn Neurodyn. doi:10.1007/s11571-013-9244-2

Wang FZ, Helian N, Wu S, Lim MG, Guo Y, Parker MA (2010) Delayed switching in memristors and memristive systems. IEEE Electron Device Lett 31(7):755–757

Wu A, Zeng Z (2012) Dynamic behaviors of memristor-based recurrent neural networks with time-varying delays. Neural Netw 36:1–10

Wu A, Zeng Z (2013) Anti-synchronization control of a class of memristive recurrent neural networks. Commun Nonlinear Sci Numer Simul 18:373–385

Wu X, Wang Y, Huang L, Zuo Y (2010) Robust stability analysis of delayed takagi-sugeno fuzzy hopfield neural networks with discontinuous activation functions. Cogn Neurodyn 4(4):347–354

Wu A, Zeng Z, Zhu X, Zhang J (2011) Exponential synchronization of memristor-based recurrent neural networks with time delays. Neurocomputing 74(17):3043–3050

Wu A, Wen S, Zeng Z (2012) Synchronization control of a class of memristor-based recurrent neural networks. Inf Sci 183(1):106–116

Yang X, Cao J (2013) Exponential synchronization of delayed neural networks with discontinuous activations. IEEE Trans Circuits Syst I 60(9):2431–2439

Yang X, Huang C, Zhang D, Long Y (2008) Dynamics of cohen–grossberg neural networks with mixed delays and impulses. Abstract Appl Anal 2008:14. Article ID 432341

Yang X, Cao J, Long Y, Rui W (2010) Adaptive lag synchronization for competitive neural networks with mixed delays and uncertain hybrid perturbations. IEEE Trans Neural Netw 21(10):1656–1667

Yang X, Zhu Q, Huang C (2011) Lag stochastic synchronization of chaotic mixed time-delayed neural networks with uncertain parameters or perturbations. Neurocomputing 74(10):1617–1625

Yang X, Wu Z, Cao J (2013) Finite-time synchronization of complex networks with nonidentical discontinuous nodes. Nonlinear Dyn 73(4):2313–2327

Zhang G, Shen Y, Sun J (2012) Global exponential stability of a class of memristor-based recurrent neural networks with time-varying delays. Neurocomputing 97:149–154

Zhang G, Shen Y, Wang L (2013a) Global anti-synchronization of a class of chaotic memristive neural networks with time-varying delays. Neural Netw 46:1–8

Zhang G, Shen Y, Yin Q, Sun J (2013b) Global exponential periodicity and stability of a class of memristor-based recurrent neural networks with multiple delays. Inf Sci 232:386–396

Zhu Q, Cao J (2010) Adaptive synchronization of chaotic cohen–crossberg neural networks with mixed time delays. Nonlinear Dyn 61(3):517–534