



Nehad Ali Shah · Azhar Ali Zafar · Shehraz Akhtar

General solution for MHD-free convection flow over a vertical plate with ramped wall temperature and chemical reaction

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Abstract The aim of the article is to study the unsteady magnetohydrodynamic-free convection flow of an electrically conducting incompressible viscous fluid over an infinite vertical plate with ramped temperature and constant concentration. The motion of the plate is a rectilinear translation with an arbitrary time-dependent velocity. Closed-form solutions for the temperature, concentration and velocity fields of the fluid are obtained. The influence of transverse magnetic field that is fixed relative either to fluid or plate is studied. Furthermore, the effects of system parameters on the fluid velocity are analyzed through numerical simulations and graphical illustrations.

Mathematics Subject Classification 76D05 · 76W05

المخلص

الهدف من هذا البحث هو دراسة تدفق حراري حر ديناميكي مغناطيسي لموصل كهربائي مائع لزج هادئ وغير قابل للضغط وذلك على صفيحة لا نهائية وعمودية خاضعة لدرجة حرارة منحدره وتركيز ثابت. فحركة الصفيحة هي انسحاب مستقيم بسرعة عشوائية مرتبطة بالزمن. لقد تم الحصول على الحلول ذات الأشكال المغلقة لدرجة الحرارة، والتركيز، وحقول السرعة للمائع. كما تمت دراسة تأثير المجال المغناطيسي المستعرض والمثبت إما بالنسبة للمائع أو للصفيحة. بالإضافة، فقد تم تحليل تأثير وسائط النظام على سرعة المائع وذلك من خلال محاكاة عددية ورسوم بيانية توضيحية.

1 Introduction

Magnetohydrodynamics (MHD) boundary layer flows of an electrically conducting fluid with heat and mass transfer have applications in engineering, geophysical environments, chemical engineering, etc. Several flow problems involving the heat and mass transfer with various mechanical, thermal, and species concentration boundary conditions have been studied. For example, Singh and Kumar [15] studied the fluctuating heat and mass transfer on unsteady MHD-free convection flow of a radiating and reacting fluid with slip on a porous

N. A. Shah (✉) · A. A. Zafar · S. Akhtar
Abdus Salam School of Mathematical Sciences, GC University, Lahore 54600, Pakistan
E-mail: nehadali199@yahoo.com

A. A. Zafar
Department of Mathematics, GC University, Lahore 54600, Pakistan



plate by considering periodic functions for the suction velocity, temperature, and concentration at the wall. The unsteady-free convection flow of a Newtonian fluid past an impulsively started infinite vertical plate in a porous medium with ramped wall temperature, ramped wall concentration, and ramped plate velocity has been recently studied by Ahmed and Dutta [1].

Ghara et al. [5] have studied the MHD-free convection flow past an impulsively moving vertical plate with ramped wall temperature. Seth et al. [14] studied Hall and Soret effects on unsteady MHD-free convection flow of radiating and chemically reactive fluid past a moving vertical plate with ramped temperature in a rotating system. The unsteady MHD-free convection flow past a porous plate under oscillatory suction velocity was analyzed by Reddy [10]. MHD-free convection flow of a second grade fluid was studied by Samiulhaq et al. in [11]. Narahari and Debnath [8] have studied the unsteady MHD-free convection flow past an accelerated vertical plate with constant heat flux and heat source. They considered two cases, namely, the magnetic lines of force held fixed relative to the fluid or held fixed relative to the plate. For more interesting and related results, see [6, 12, 13, 16] and the references therein.

In the present paper, the unsteady MHD-free convection flow near a vertical plate is considered. The plate has a translational motion in its plane with an arbitrary time-dependent velocity. The wall temperature varies as a ramped law and the wall concentration is constant. The heat generation or absorption and the chemical reaction are also considered. The fluid is electrically conducting and regarding the applied magnetic field two cases are considered, namely, when the magnetic lines of force are held fixed to the fluid or to the plate [8]. The difference between fluid velocities in the two cases is studied and some properties are highlighted. The governing linear partial differential equations are written into a non-dimensional form and solved by means of the Laplace transform method. The influence of the system parameters (e.g., magnetic parameter, Grashof numbers, chemical reaction, and heat absorption parameters) on the fluid velocity are also analyzed through graphical illustrations.

2 Mathematical formulation of the problem

Consider the unsteady-free convection flow of an incompressible viscous, electrically conducting fluid. The fluid is near an infinite vertical plate with ramped temperature and constant concentration. The motion of the plate is a rectilinear translation with an arbitrary time-dependent velocity. We introduce a coordinate system with x -axis along the plate in vertical upward direction and y -axis normal to the plate. A uniform transverse magnetic field of strength B_o is applied. Initially, at time $t = 0$, the plate and the fluid are at rest with the same temperature T_∞ and species concentration C_∞ .

After time $t = 0^+$, the plate moves with the velocity $U_o f(t)$ in its own plane along the x -axis. Here, U_o is a constant velocity and $f(\cdot)$ is a dimensionless piecewise continuous function, whose value $f(0) = 0$. Heat is supplied to the plate as a time-ramped function in the presence of heat source and chemical reaction. The species concentration at the plate is constant C_w . We further assume that:

- 1°. The magnetic Reynolds number is small, so that the induced magnetic field is negligible in comparison with the applied magnetic field.
- 2°. Viscous dissipation, radiative effects, and Joule heating terms are neglected in the energy equation (usually in free convection flows the velocity has small values). However, according to Magyari and Pantokratoras [7], the radiative effects can be easily included by a simple rescaling of the Prandal number.
- 3°. No external electric field is applied and the effect of polarization of ionized fluid is negligible; therefore, electric field is assume to be zero.
- 4°. There exists a first-order chemical reaction between the fluid and species concentration. The level of species concentration is very low, so that the heat generated by chemical reaction can be neglected.

Since the plate is infinite extended in x and z directions, therefore, all physical quantities are functions of the spatial coordinate y and time t only.

Under usual Boussinesq's approximation, the flow is governed by the following system of equations [8, 15]:

$$\frac{\partial u(y, t)}{\partial t} = \nu \frac{\partial^2 u(y, t)}{\partial y^2} + g\beta_T(T(y, t) - T_\infty) + g\beta_C(C(y, t) - C_\infty) - \frac{\sigma B_o^2}{\rho} u(y, t), \quad (1)$$

$$\rho C_p \frac{\partial T(y, t)}{\partial t} = k \frac{\partial^2 T(y, t)}{\partial y^2} - Q[T(y, t) - T_\infty], \quad (2)$$



$$\frac{\partial C(y, t)}{\partial t} = D_m \frac{\partial^2 C(y, t)}{\partial y^2} - R[C(y, t) - C_\infty]. \tag{3}$$

Into above equations, $u(y, t)$, $T(y, t)$, and $C(y, t)$ are velocity, temperature, and species concentration of the fluid, respectively, ν is the kinematic viscosity, g is the acceleration due to gravity, β_T is the thermal expansion coefficient, ρ is fluid density, β_C is the volumetric coefficient of expansion with species concentration, σ is electrical conductivity, C_p is the specific heat at constant pressure, k is the thermal conductivity, Q is the heat generation or absorption coefficient, D_m is the chemical molecular diffusivity, and R is chemical reaction parameter.

Equation (1) is valid when the magnetic lines of force are fixed relative to the fluid. If the magnetic field is fixed relative to the plate, the momentum Eq. (1) is replaced by [8,17]

$$\begin{aligned} \frac{\partial u(y, t)}{\partial t} = & \nu \frac{\partial^2 u(y, t)}{\partial y^2} + g\beta_T(T(y, t) - T_\infty) + g\beta_C(C(y, t) - C_\infty) \\ & - \frac{\sigma B_o^2}{\rho} [u(y, t) - U_o f(t)], \end{aligned} \tag{4}$$

Equations (1) and (4) can be combined as

$$\frac{\partial u(y, t)}{\partial t} = \nu \frac{\partial^2 u(y, t)}{\partial y^2} + g\beta_T(T(y, t) - T_\infty) + g\beta_C(C(y, t) - C_\infty) - \frac{\sigma B_o^2}{\rho} [u(y, t) - \varepsilon U_o f(t)], \tag{5}$$

where

$$\varepsilon = \begin{cases} 0, & \text{if } B_o \text{ is fixed relative to the fluid;} \\ 1, & \text{if } B_o \text{ is fixed relative to the plate.} \end{cases}$$

The appropriate initial and boundary conditions are

$$u(y, 0) = 0, \quad T(y, 0) = T_\infty, \quad C(y, 0) = C_\infty, \quad y \geq 0, \tag{6}$$

$$u(0, t) = U_o f(t), \quad T(0, t) = \begin{cases} T_\infty + (T_w - T_\infty) \frac{t}{t_o}, & 0 < t \leq t_o; \\ T_w, & t > t_o. \end{cases}, \quad C(0, t) = C_w, \tag{7}$$

$$u(y, t) < \infty, \quad T(y, t) \rightarrow T_\infty, \quad C(y, t) \rightarrow C_\infty \quad \text{as } y \rightarrow \infty. \tag{8}$$

Introducing the dimensionless variables

$$\begin{aligned} y^* = \frac{y}{\sqrt{\nu t_o}}, \quad t^* = \frac{t}{t_o}, \quad u^* = \frac{u}{U_o}, \quad T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad C^* = \frac{C - C_\infty}{C_w - C_\infty}, \\ G_r = \frac{g\beta_T t_o (T_w - T_\infty)}{U_o}, \quad G_m = \frac{g\beta_C t_o (T_w - T_\infty)}{U_o}, \quad M = \sqrt{\nu t_o} B_o \sqrt{\frac{\sigma}{\mu}}, \\ Pr = \frac{\mu C_p}{k}, \quad Q^* = \frac{Q t_o}{\rho C_p}, \quad Sc = \frac{\nu}{D_m}, \quad R^* = R t_o, \quad f^*(t^*) = f(t_o t^*) \end{aligned} \tag{9}$$

and dropping out the star notation, we get

$$\frac{\partial u(y, t)}{\partial t} = \frac{\partial^2 u(y, t)}{\partial y^2} + G_r T(y, t) + G_m C(y, t) - M^2(u(y, t) - \varepsilon f(t)), \tag{10}$$

$$\frac{\partial T(y, t)}{\partial t} = \frac{1}{Pr} \frac{\partial^2 T(y, t)}{\partial y^2} - Q T(y, t), \tag{11}$$

$$\frac{\partial C(y, t)}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C(y, t)}{\partial y^2} - R C(y, t), \tag{12}$$

$$u(y, 0) = 0, \quad T(y, 0) = 0, \quad C(y, 0) = 0, \tag{13}$$

$$u(0, t) = f(t), \quad T(0, t) = \begin{cases} t, & 0 \leq t \leq 1; \\ 1, & t > 1. \end{cases} = tH(t) - (t - 1)H(t - 1), \quad C(0, t) = 1, \tag{14}$$

$$u(y, t) < \infty, \quad T(y, t) \rightarrow 0, \quad C(y, t) \rightarrow 0 \quad \text{as } y \rightarrow \infty. \tag{15}$$

Into above relations, G_r , G_m , M , Pr , Q , R , and Sc denote thermal Grashof number, Grashof number, Hartman number, Prandtl number, the dimensionless heat generation or absorption coefficient, the dimensionless chemical reaction parameter, and the Schmidt number, respectively, whereas

$$H(t) = \begin{cases} 0, & t \leq 0; \\ 1, & t > 0 \end{cases}$$

is the Heaviside unit step function.

3 Solution of the problem

In the following, the solution of partial differential equations (10)–(12) with initial and boundary conditions (13)–(15) will be determined by Laplace transform. The momentum equation depends upon energy and concentration equation; therefore, we shall first establish the exact solutions for temperature and concentration.

3.1 Temperature field

In a recent paper, Ghara et al. [5] studied a similar MHD-free convection flow with thermal radiation and ramped wall temperature but without heat source. Due to their assumption regarding the relative heat flux, the dimensionless form of the energy equation is identical to our Eq. (11) with the radiation parameter Ra instead of Q . As the corresponding initial and boundary conditions are also identical to our conditions (14)₂ and (15)₂, we directly present the temperature field:

$$T(y, t) = H(t)\Phi(y, t; Pr, Q) - H(t - 1)\Phi(y, t - 1; Pr, Q), \quad (16)$$

and its Laplace transform [2]

$$\bar{T}(y, s) = \frac{1 - e^{-s}}{s^2} e^{-y\sqrt{Pr(s+Q)}}, \quad (17)$$

to be used later for velocity. The function $\Phi(y, t; a, b)$, defined by

$$\begin{aligned} \Phi(y, t; a, b) = & \frac{2t\sqrt{b} - y\sqrt{a}}{4\sqrt{b}} e^{-y\sqrt{ab}} \operatorname{erfc}\left(\frac{y\sqrt{a}}{2\sqrt{t}} - \sqrt{bt}\right) \\ & + \frac{2t\sqrt{b} + y\sqrt{a}}{4\sqrt{b}} e^{y\sqrt{ab}} \operatorname{erfc}\left(\frac{y\sqrt{a}}{2\sqrt{t}} + \sqrt{bt}\right), \end{aligned} \quad (18)$$

is identical to that of Ghara et al. [5, Eq. (21)] for $Q = Ra$.

3.2 Species concentration

Applying the Laplace transform to Eq. (12) and using the corresponding initial and boundary conditions, we find that

$$\frac{\partial^2 \bar{C}(y, s)}{\partial y^2} - Sc(s + R)\bar{C}(y, s) = 0; \quad \bar{C}(0, s) = \frac{1}{s}, \quad \bar{C}(y, s) \rightarrow 0 \quad \text{as } y \rightarrow 0, \quad (19)$$

where $\bar{C}(y, s)$ is the Laplace transform of $C(y, t)$. The solution of the differential equation (19)₁ with the adjoining boundary conditions is

$$\bar{C}(y, s) = \frac{1}{s} e^{-y\sqrt{Sc\sqrt{s+R}}}. \quad (20)$$

By now applying the inverse Laplace transform to Eq. (20), we can easily obtain the concentration distribution and is known in the literature.



3.3 Calculation of fluid velocity

Applying the Laplace transform to Eq. (10), and using the initial and boundary conditions (13)₁, (14)₁ and (15)₁ as well as Eqs. (17), (20), we find that

$$\frac{\partial^2 \bar{u}(y, s)}{\partial y^2} - (s + M^2)\bar{u}(y, s) = -G_r \frac{1 - e^{-s}}{s^2} e^{-y\sqrt{Pr}\sqrt{s+Q}} - G_m \frac{1}{s} e^{-y\sqrt{Sc}\sqrt{s+R}} - \varepsilon M^2 F(s), \tag{21}$$

$$\bar{u}(0, s) = F(s), \quad \bar{u}(y, s) < \infty \quad \text{as } y \rightarrow \infty, \tag{22}$$

where $\bar{u}(y, s)$ and $F(s)$ are the Laplace transforms of $u(y, t)$ and $f(t)$, respectively. The solution of the problem (21), (22), is

$$\begin{aligned} \bar{u}(y, s) = & F(s)e^{-y\sqrt{s+M^2}} + \varepsilon M^2 F(s) \frac{1 - e^{-y\sqrt{s+M^2}}}{s + M^2} + \frac{G_r(1 - Pr)}{(QPr - M^2)^2} (1 - e^{-s}) \left[m \frac{e^{-y\sqrt{Pr}\sqrt{s+Q}}}{s^2} \right. \\ & \left. - m \frac{e^{-y\sqrt{s+M^2}}}{s^2} - \frac{e^{-y\sqrt{Pr}\sqrt{s+Q}}}{s} + \frac{e^{-y\sqrt{s+M^2}}}{s} + \frac{e^{-y\sqrt{Pr}\sqrt{s+Q}}}{s+m} - \frac{e^{-y\sqrt{s+M^2}}}{s+m} \right] \\ & - \frac{G_m}{RSc - M^2} \left[\frac{e^{-y\sqrt{Sc}\sqrt{s+R}}}{s} - \frac{e^{-y\sqrt{s+M^2}}}{s} - \frac{e^{-y\sqrt{Sc}\sqrt{s+R}}}{s+n} + \frac{e^{-y\sqrt{s+M^2}}}{s+n} \right], \end{aligned} \tag{23}$$

where

$$m = \frac{QPr - M^2}{Pr - 1}, \quad n = \frac{RSc - M^2}{Sc - 1} \quad \text{for } Pr \neq 1 \text{ and } Sc \neq 1.$$

Applying the inverse Laplace transforms to Eq. (23), we find that

$$u(y, t) = u_m(y, t) + u_T(y, t) + u_C(y, t), \tag{24}$$

where

$$u_m(y, t) = \frac{y}{2\sqrt{\pi}} \int_0^t \frac{f(t - \tau)}{\tau\sqrt{\tau}} e^{\left(-\frac{y^2}{4\tau} - M^2\tau\right)} d\tau + \varepsilon M^2 \int_0^t f(t - \tau) e^{-M^2\tau} \operatorname{erf}\left(\frac{y}{2\sqrt{\tau}}\right) d\tau, \tag{25}$$

$$u_T(y, t) = \frac{G_r(Pr - 1)}{(QPr - M^2)} \{H(t)W(y, t) - H(t - 1)W(y, t - 1)\}, \tag{26}$$

$$\begin{aligned} u_C(y, t) = & -\frac{G_m}{RSc - M^2} \{\Psi(y, t; 0, R, Sc) - \Psi(y, t; 0, R, 1) - \Psi(y, t; -n, R, Sc) \\ & + \Psi(y, t; -n, R, 1)\}. \end{aligned} \tag{27}$$

denotes that the mechanical, thermal, and concentration components of velocity.

Into above relations

$$\begin{aligned} W(y, t) = & m\Phi(y, t; Pr, Q) - m\Phi(y, t; 1, M^2) - \Psi(y, t; 0, Q, Pr) + \Psi(y, t; 0, M^2, 1) \\ & + \Psi(y, t; -m, Q, Pr) - \Psi(y, t; -m, M^2, 1), \end{aligned} \tag{28}$$

where as the functions $\Psi(y, t; a, b, c)$ as defined in Appendix (A.3) of [4]

$$\begin{aligned} \Psi(y, t; a, b, c) = & L^{-1} \left\{ \frac{e^{-y\sqrt{c}\sqrt{s+b}}}{s - a} \right\} \\ = & \frac{e^{at}}{2} \left[e^{-y\sqrt{c}\sqrt{a+b}} \operatorname{erfc} \left(\frac{y\sqrt{c}}{2\sqrt{t}} - \sqrt{(a+b)t} \right) \right. \\ & \left. + e^{y\sqrt{c}\sqrt{a+b}} \operatorname{erfc} \left(\frac{y\sqrt{c}}{2\sqrt{t}} + \sqrt{(a+b)t} \right) \right]. \end{aligned} \tag{29}$$

Equation (25) seems to not satisfy the boundary condition (14)₁. To eliminate this drawback, we rewrite (25) in the equivalent form:

$$u_m(y, t) = \frac{2}{\sqrt{\pi}} \int_{\frac{y}{2\sqrt{t}}}^{\infty} f\left(t - \frac{y^2}{4s^2}\right) e^{\left(-s^2 - \frac{M^2 y^2}{4s^2}\right)} ds + \varepsilon M^2 \int_0^t f(t - s) e^{-M^2 s} \operatorname{erf}\left(\frac{y}{2\sqrt{s}}\right) ds. \quad (30)$$

Moreover, to compare the fluid velocities corresponding to the two ways of implementing magnetic field, we note that the two velocities differ through the term:

$$V(y, t) = M^2 \int_0^t f(t - \tau) e^{-M^2 \tau} \operatorname{erf}\left(\frac{y}{2\sqrt{\tau}}\right) d\tau. \quad (31)$$

Now, it is important to highlight the following aspects:

- (1) If $f(\cdot)$ is a piecewise continuous and positive function, $V(y, t)$ has positive values. Consequently, for the magnetic field is fixed relative to the plate, the fluid velocity is larger than the fluid velocity corresponding to the case when the magnetic field is fixed to the fluid. An opposite trend appears if the function $f(\cdot)$ is negative.
- (2) The term $V(y, t)$ is an increasing function of the spatial variable y . Therefore, the fluid velocity is significantly changed in the neighborhood of the plate if the magnetic field is fixed relative to the plate.
- (3)

$$\begin{aligned} \lim_{y \rightarrow \infty} u(y, t) &= L^{-1} \left\{ \lim_{y \rightarrow \infty} \bar{u}(y, s) \right\} = L^{-1} \left\{ \varepsilon M^2 \frac{F(s)}{s + M^2} \right\} \\ &= \varepsilon M^2 \int_0^t f(t - \tau) e^{-M^2 \tau} d\tau = h(t) = \begin{cases} 0, & \varepsilon = 0 \\ M^2 \int_0^t f(t - \tau) e^{-M^2 \tau} d\tau, & \varepsilon = 1 \end{cases} \end{aligned}$$

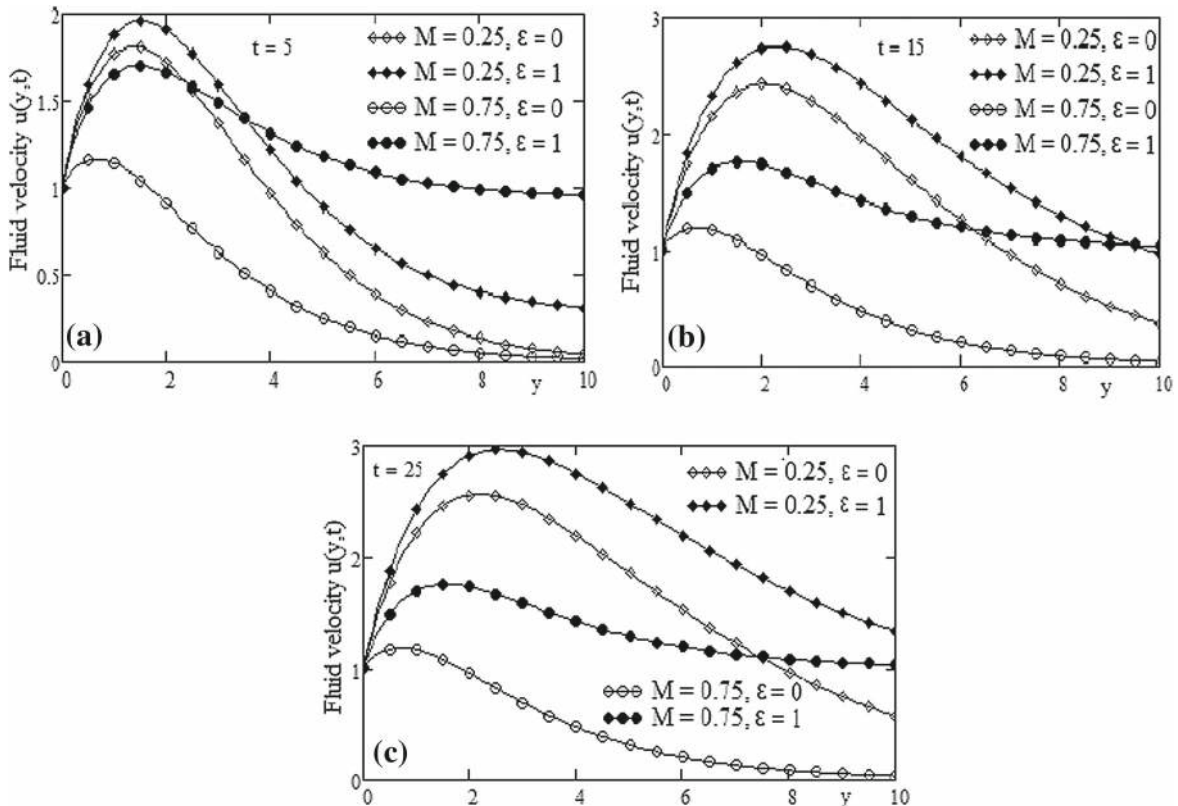


Fig. 1 Profiles of the dimensionless velocity given by Eq. (25) for $Pr = 0.71$, $Q = 0.6$, $Sc = 0.2$, $R = 0.7$, $Gr = 1.25$, $Gm = 0.5$, $M = 0.25$, and 0.75 and different values of time t

Consequently, unlike the case when the magnetic field is fixed to the fluid, the fluid at infinity does not remain at rest if the magnetic field is fixed relative to the plate.

4 Special cases

As thermal and concentration components of velocity do not depend on the plate motion. However, heat and mass transfer can influence the fluid motion and we have to know if their influence is significant or it can be neglected in some motions with possible engineering applications.

4.1 Motion of the plate with constant velocity ($f(t) = H(t)$)

We take $f(t) = H(t)$ (the Heaviside unit step function) into Eq. (25). After lengthy but straightforward computation, the mechanical component take the form:

$$u_m(y, t) = (1 - \varepsilon)\Psi(y, t; 0, M^2, 1) + \varepsilon \left[1 - e^{-M^2 t} \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) \right]. \tag{32}$$

As it was to be expected the corresponding results are identical to those obtained by Narahari and Debnath [8, Eq. (11a) with $a_o = 0$] and Tokis [17, Eq. (12)].

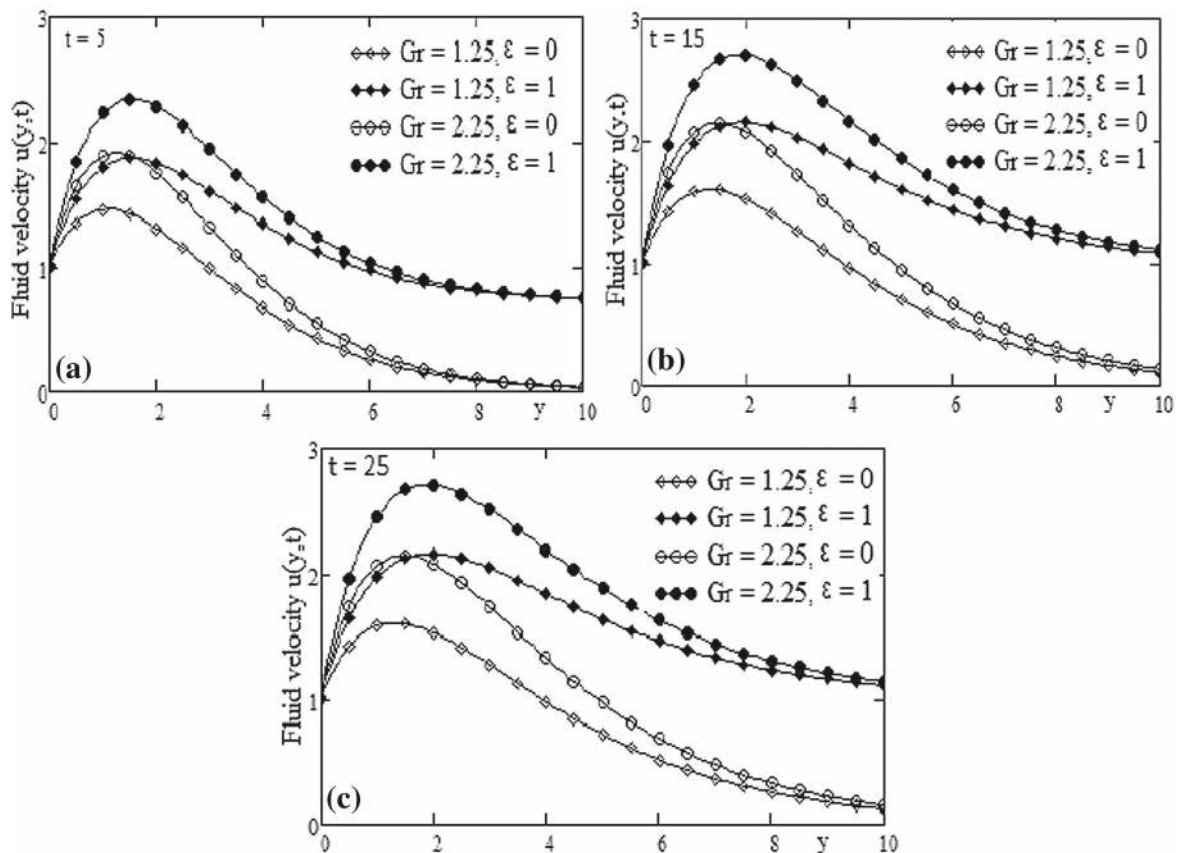


Fig. 2 Profiles of the dimensionless velocity given by Eq. (25) for $Pr = 0.71, Q = 0.6, Sc = 0.2, R = 0.7, M = 0.25, Gm = 0.5, Gr = 1.25$ and 2.25 and different values of time t

4.2 Motion of the plate with exponential acceleration ($f(t) = H(t)e^{bt}$)

Introducing $f(t) = H(t)e^{bt}$ into Eq. (25), we find that

$$u_m(y, t) = \left(1 - \frac{\varepsilon M^2}{b + M^2}\right) \Psi(y, t; b, M^2, 1) + \frac{\varepsilon M^2}{b + M^2} \left[e^{bt} - e^{-M^2 t} \operatorname{erf}\left(\frac{y}{2\sqrt{t}}\right) \right], \tag{33}$$

where

$$L^{-1} \left\{ \frac{e^{-y\sqrt{s+M^2}}}{s-b} \right\} = \Psi(y, t; b, M^2, 1), \tag{34}$$

$$L^{-1} \left\{ \frac{e^{-y\sqrt{s+M^2}} - 1}{(s-b)(s+M^2)} \right\} = \frac{1}{b+M^2} \Psi(y, t; b, M^2, 1) - \frac{1}{b+M^2} \left[e^{bt} - e^{-M^2 t} \operatorname{erf}\left(\frac{y}{2\sqrt{t}}\right) \right], \tag{35}$$

The results are identical to those obtained by Pattnaik et al. [9, Eq. (13)] with $a = b$, $\lambda = M^2$, $\frac{1}{K_p} = 0$ and $\gamma = 0$ in the absence of thermal and concentration effects and for the case when magnetic field is held fixed relative to the fluid.

Indeed, assigning to $f(\cdot)$ suitable forms, we can determine exact solutions for any motion with technical relevance of this type.

5 Numerical results and discussion

The unsteady-free convection flow of an incompressible viscous, electrically conducting fluid is studied. The fluid is near an infinite vertical plate with ramped temperature and constant concentration. The motion

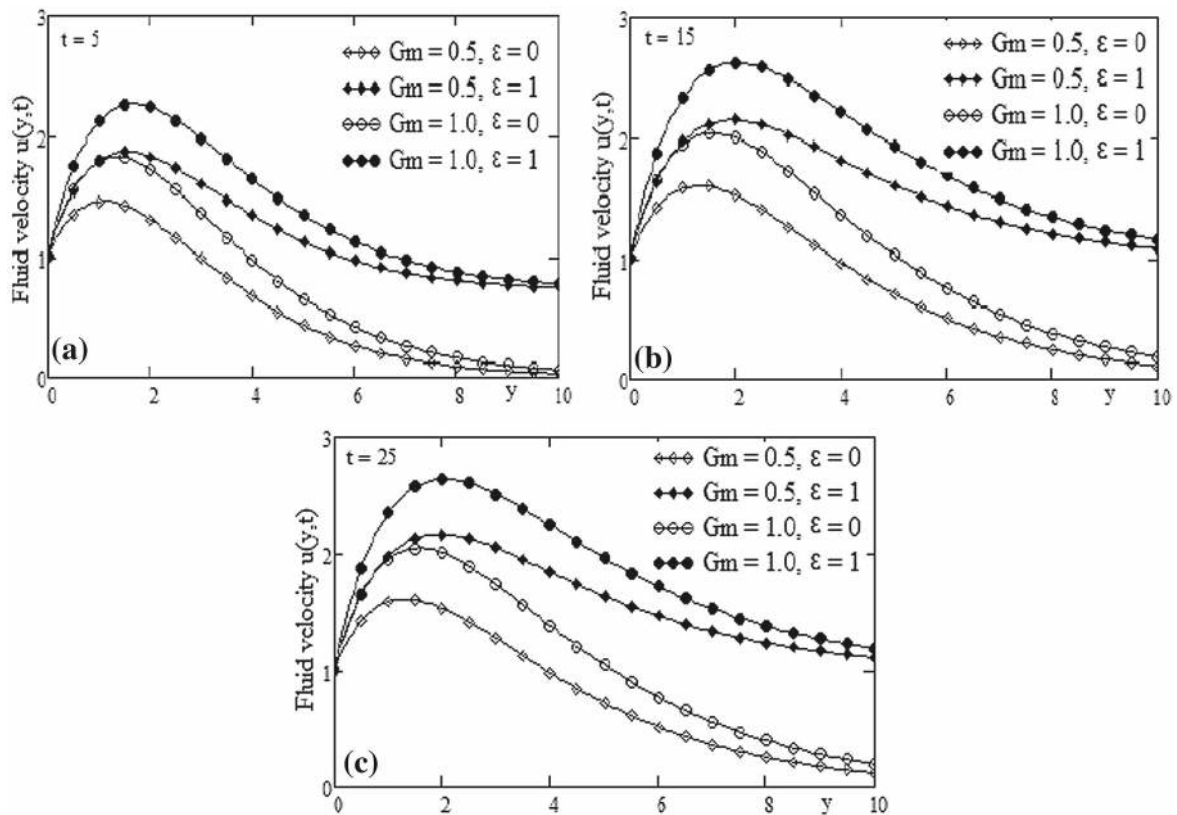


Fig. 3 Profiles of the dimensionless velocity given by Eq. (25) for $Pr = 0.71$, $Q = 0.6$, $Sc = 0.2$, $R = 0.7$, $Gr = 1.25$, $M = 0.25$, $Gm = 0.5$ and 1 and different values of time t

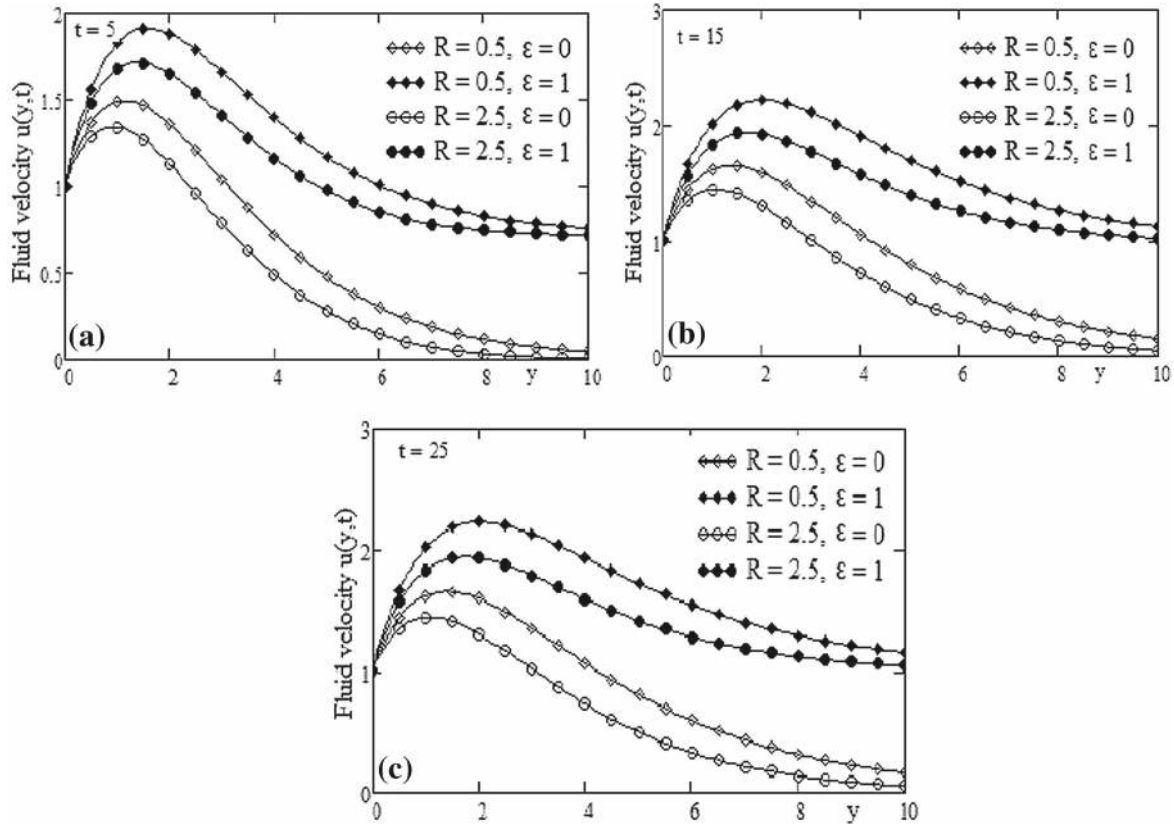


Fig. 4 Profiles of the dimensionless velocity given by Eq. (25) for $Pr = 0.71$, $Q = 0.6$, $Sc = 0.2$, $M = 0.25$, $Gr = 1.25$, $Gm = 0.5$, $R = 0.5$ and 2.5 and different values of time t

of the plate is a rectilinear translation with an arbitrary time-dependent velocity. Closed-form solutions for the dimensionless temperature, concentration, and velocity fields of the fluid are obtained. The influence of transverse magnetic field that is fixed relative to fluid or plate is studied. It is important to note that the influence of the flow parameters is significantly different on fluid flow, depending on how the magnetic lines of force are fixed relative to the fluid or the plate. Finally, two particular cases for the translational motion of the plate, namely, the translation with constant velocity, respectively, exponential accelerated motion are considered. Numerical results for velocity have been computed for several values of magnetic parameter M , Grashof number G_r , mass Grashof number G_m , chemical reaction parameter R , and heat source parameter Q .

The velocity profiles versus the spatial variable y at time $t = 15$ and for constant plate velocity ($f(t) = H(t)$) are shown in Figs. 1, 2, 3, 4, and 5. The velocity profiles $u(y, t)$, are plotted when the magnetic field is being fixed to the fluid ($\epsilon = 0$) and to the moving plate ($\epsilon = 1$).

The influence of magnetic field M on the velocity profiles is presented in Fig. 1. It is known that, under the influence of a magnetic field on an electrically conducting fluid, a resistive force arises (so called the Lorentz force). This force has tendency to slow down the fluid motion in the boundary layer. It can be seen from Fig. 1 that the fluid velocity decreases as the magnetic field M increases. In addition, it is noted that if the magnetic field is fixed to the fluid ($\epsilon = 0$), the values of the fluid velocity are lower than in the case when the magnetic field is fixed to the plate ($\epsilon = 1$).

The effects of the thermal Grashof number G_r and the mass Grashof number G_m on the fluid velocity are shown in Figs. 2 and 3. For both parameters, velocity has a maximum value in the vicinity of the plate and tends towards the value $h(t)$ for large values of the spatial coordinate y . It is further observed that the values of the fluid velocity increase with increasing values of G_r and G_m . The influence of the dimensionless chemical reaction parameter R on the fluid velocity is presented in Fig. 4. and it is observed that the fluid velocity decreases with increasing values of the parameter R .

In Fig. 5, we have plotted the velocity profiles versus y for two values of the heat absorption parameter Q . It can be seen from these curves that the velocity decreases with increasing of the values of parameter Q .

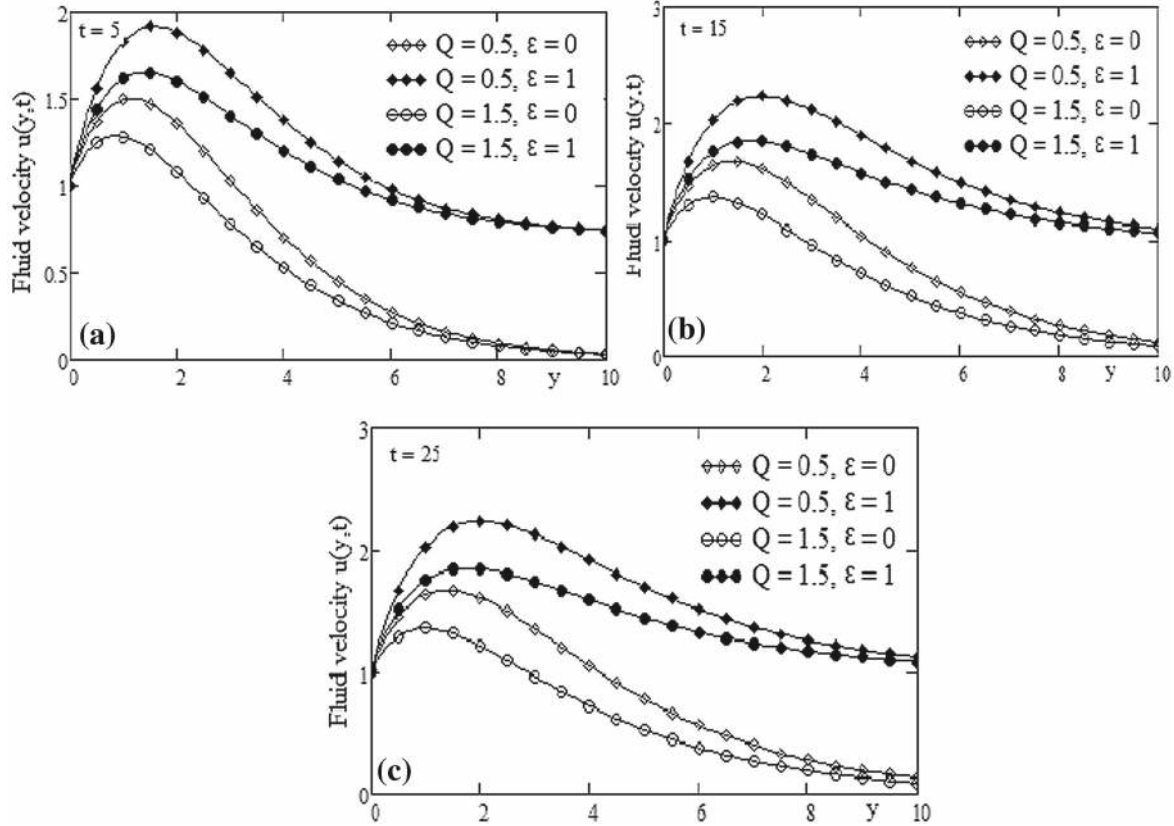


Fig. 5 Profiles of the dimensionless velocity given by Eq. (25) for $Pr = 0.71, M = 0.25, Sc = 0.2, R = 0.7, Gr = 1.25, Gm = 0.5, Q = 0.5$ and 1.5 and different values of time t

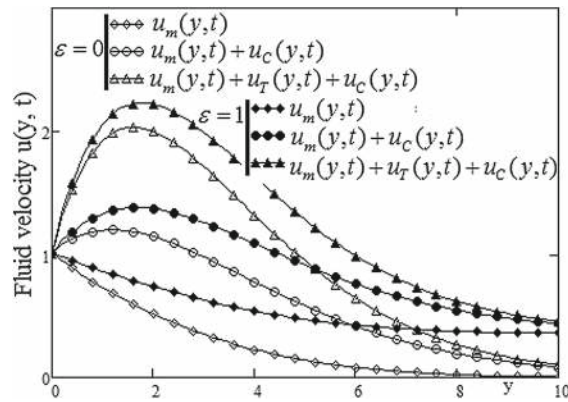


Fig. 6 Profiles of the dimensionless velocities $u_m, u_m + u_C$ and $u_m + u_T + u_C$ for $Pr = 0.71, M = 0.25, Sc = 0.2, R = 0.7, Gr = 1.25, Gm = 0.5, Q = 0.6$ and different values of time t

In Fig. 6, we have plotted velocities $u_m(y, t), u_m(y, t) + u_C(y, t)$ and $u_m(y, t) + u_C(y, t) + u_T(y, t)$ versus y to investigate the contributions of mechanical, thermal, and concentration components of velocity on the fluid motion. It is observed that contributions of mechanical, thermal, and concentration components of velocity on the fluid motion are significant and they cannot be neglected.

6 Conclusions

The unsteady-free convection flow of an incompressible viscous and electrically conducting fluid is studied. The fluid is near an infinite vertical plate with ramped temperature and constant concentration. Closed-form solutions for dimensionless temperature, concentration, and velocity field of the fluid are obtained. The influence of transverse magnetic field that is held fixed relative to fluid or to the plate is studied.

The motion of the plate is a rectilinear translation with an arbitrary time-dependent velocity $f(\cdot)$; assigning to this $f(\cdot)$ suitable forms, we can determine exact solutions for any motion with technical relevance of this type. The plate temperature changes as a time-ramped function and the concentration on the plate is constant. The resulting coupled partial differential equations are written in the non-dimensional form and are solved by means of the Laplace transforms.

Through graphical illustrations for the case when plate is moving with uniform velocity, the influence of magnetic field and of the system parameters on the fluid velocity is brought to light. Some useful conclusions of the study are as under:

- If the magnetic field is fixed relative to the moving plate, the fluid velocity differs significantly from the case when the magnetic field is fixed relative to the fluid.
- At large distances from the plate, the fluid will be at rest when the magnetic field is fixed relative to the fluid, but it attains a finite non zero value if the magnetic field is fixed relative to the plate.
- For increasing values of the Hartmann number, the fluid velocity decreases; therefore, stronger magnetic field leads to slower flows.
- The fluid velocity increases with increasing values of the thermal Grashof and mass Grashof number and decreases with respect to the increasing values of chemical reaction and heat absorption parameters R and Q .
- Contributions of mechanical, thermal, and concentration components of velocity on the fluid motion are significant and they cannot be neglected.

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