Joint optimization of a partially coherent Gaussian beam for free-space optical communication over turbulent channels with pointing errors

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Received November 5, 2012; revised December 27, 2012; accepted December 27, 2012; posted January 2, 2013 (Doc. ID 179210); published January 28, 2013

Joint beam width and spatial coherence length optimization is proposed to maximize the average capacity in partially coherent free-space optical links, under the combined effects of atmospheric turbulence and pointing errors. An optimization metric is introduced to enable feasible translation of the joint optimal transmitter beam parameters into an analogous level of divergence of the received optical beam. Results show that near-ideal average capacity is best achieved through the introduction of a larger receiver aperture and the joint optimization technique.

OCIS codes: 010.1300, 010.1330, 010.3310, 060.2605.

Free-space optical (FSO) communication is an emerging low-cost, high-bandwidth access solution, albeit one hampered by the combined effects of atmospheric loss, turbulence, and pointing errors (PEs) [1,2]. The use of spatially partially coherent Gaussian beams through random media has been substantially explored already [3,4] and has revealed the benefits of mitigating the scintillations and PEs. This inevitably causes a reduction in the mean received intensity, thus implying the necessity of partially coherent beam (PCB) optimization.

In [5], the problem of maximizing the mean received intensity has been examined, showing that a fully coherent beam optimizes the intensity, whereas a PCB has the advantage of mitigating the scintillation effect. In [6], an optimization criterion for the laser initial coherence has been proposed. The problem of coherence length optimization was investigated by considering a heuristic metric in [7] and the outage probability in [8]. Nevertheless, the effects of PEs and aperture averaging were not considered in these works. In [9], a theoretical model has been proposed, showing that the beam waist and divergence angle can be tuned to optimize the average capacity over turbulence channels with PEs. However, the degree of spatial coherence was not taken into account in their beam model. Cang and Liu [10] studied the achievable average capacity for PCBs through non-Kolmogorov turbulence without considering any optimization. In this Letter, we propose joint beam width and coherence length optimization to maximize the average capacity in partially coherent Gaussian FSO systems, under the influence of turbulence and PEs, while taking into account the aperture-averaging effect. In particular, an optimization metric is developed to determine the optimum beam divergence at the receiver.

We consider a horizontal FSO link (Fig. 1), employing intensity modulation with direct detection. The information signal is modulated onto the instantaneous intensity of a lowest-order Gaussian-beam wave [11], which then passes through a phase diffuser, thus altering the beam divergence while retaining its beam like properties [4]. The optical signal is collected by a circular Gaussian lens of diameter D [2,11] and focused onto a photodetector, to produce a photocurrent $y_\kappa = h\gamma x_\kappa + n_o$, where γ is the detector responsivity, $x_\kappa \in \{0, 2P_{\mathrm{FSO}}\}$ is the optical signal intensity, P_{FSO} is the average transmitted optical power, and n_o is additive white Gaussian noise with variance σ_n^2 [2]. The channel fading coefficient can be described as $h = h_l h_s h_p$, where h_l, h_s and h_p denote the attenuation, scintillation, and PEs, respectively. The probability density function of h is given by [2]

$$f_h(h) = \frac{\xi^2}{(A_0 h_l)^{\xi^2}} h^{\xi^2 - 1} \int_{h/A_0 h_l}^{\infty} h_s^{-\xi^2} f_{h_s}(h_s) \mathrm{d}h_s, \qquad (1)$$

where $f_{h_s}(h_s)$ is modeled using the log-normal and gamma–gamma distributions for the weak and strong turbulence, respectively [2,11]. $\xi = w_{z_{\mathrm{eq}}}/2\sigma_{\mathrm{pe}}$ is the ratio between the equivalent receiver beam width and the PE displacement (jitter) standard deviation, $A_0 = [\mathrm{erf}(v)]^2$, $v = \sqrt{\pi} D/(2\sqrt{2} w_L)$, $w_{z_{\mathrm{eq}}} = w_L[\sqrt{\pi}\,\mathrm{erf}(v)/(2v\exp(-v^2))]^{1/2}$ [2]. We assume homogeneous turbulence along the propagation path [11]. The received electrical signal-to-noise ratio (SNR) is defined as $\mathrm{SNR}(h) = 2P_{\mathrm{FSO}}^2 \gamma^2 h^2/\sigma_n^2$, which is a fluctuating term due to the influence of h [2].

We consider a Gaussian Schell beam model [4] with an effective beam radius w_0. At link distance L, the receiving beam size is $w_L = w_0(\Theta_n^2 + \zeta\Lambda_n^2)^{1/2}$, where $\Theta_n = 1 - (L/F_0)$ and $\Lambda_n = 2L/(kw_0^2)$; F_0 is the phase front

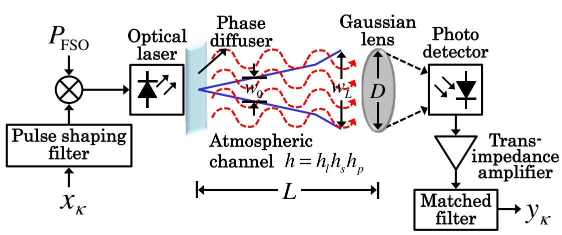

Fig. 1. (Color online) Block diagram of a horizontal FSO link.

radius of curvature at the transmitter, $k = 2\pi/\lambda$ is the optical wave number, and λ is the laser wavelength. The global coherence parameter $\zeta = \zeta_s + (2w_0^2/\rho_0^2)$ is related to the source coherence parameter $\zeta_s = 1 + (2w_0^2/l_c^2)$ and the coherence length of a spherical wave $\rho_0 = (0.55C_n^2k^2L)^{-3/5}$, where l_c is the spatial coherence length and C_n^2 is the refractive-index structure parameter of the atmosphere. The point-receiver scintillation index is given by [4,8,11]

$$\sigma_I^2(0) \cong 4.42\sigma_R^2\Lambda_L^{5/6}\frac{\sigma_{pe}^2}{w_L^2} + 3.86\sigma_R^2\left\{0.40[(1+2\Theta_L)^2 + 4\Lambda_L^2]^{5/12} \times \cos\left[\frac{5}{6}\tan^{-1}\left(\frac{1+2\Theta_L}{2\Lambda_L}\right)\right] - \frac{11}{16}\Lambda_L^{5/6}\right\}, \quad (2)$$

where $\sigma_R^2 = 1.23C_n^2k^{7/6}L^{11/6}$ is the Rytov variance for a plane wave, $\Theta_L = 1 + (L/F_L)$ and $\Lambda_L = 2L/(kw_L^2)$. The phase front radius of curvature at the receiver is $F_L = L(\Theta_n^2 + \zeta\Lambda_n^2)/(\phi\Lambda_n - \zeta\Lambda_n^2 - \Theta_n^2)$, with $\phi \equiv (\Theta_n/\Lambda_n) - (\Lambda_n w_0^2/\rho_0^2)$ [3,4]. The aperture-averaging effect is taken into account in the aperture-averaged scintillation index $\sigma_I^2(D) = A_G\sigma_I^2(0)$, where the aperture-averaging factor A_G is given by [4]

$$A_G = \frac{16}{\pi}\int_0^1 \delta d\delta \exp\left\{\frac{-D^2\delta^2}{\rho_0^2}\left(2 + \frac{\rho_0^2}{w_0^2\Lambda_n^2} - \frac{\rho_0^2\phi^2}{w_L^2}\right)\right\} \times \left[\cos^{-1}(\delta) - \delta\sqrt{1-\delta^2}\right]. \quad (3)$$

In the case of an unknown channel at the receiver, the average capacity is expressed as [12]

$$\langle C\rangle = \sum_{x=0}^{1} P_X(x)\int_{-\infty}^{\infty} f(y|x) \times \log_2\left[\frac{f(y|x)}{\sum_{m=0,1} f(y|x=m)P_X(m)}\right]dy, \quad (4)$$

where $P_X(x=0) = P_X(x=1) = 0.5$ is the probability of transmission of a bit, and $f(y|x)$ is given by

$$f(y|x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_n^2}}\exp\left[-\frac{y^2}{2\sigma_n^2}\right], & x=0 \\ \frac{1}{\sqrt{2\pi\sigma_n^2}}\int_0^\infty \exp\left[-\frac{(y-2P_{\mathrm{FSO}}\gamma h)^2}{2\sigma_n^2}\right]f_h(h)dh, & x=1 \end{cases}. \quad (5)$$

Figure 2(a) depicts $\langle C\rangle$ against the average electrical SNR $\langle \mathrm{SNR}\rangle$ for a range of l_c, under a weak turbulence condition with $C_n^2 = 3.1230 \times 10^{-16}\ \mathrm{m}^{-2/3}$ ($\sigma_R^2 = 0.25$). The variation of $\langle C\rangle$ with respect to l_c at $\langle \mathrm{SNR}\rangle = 14$ dB (corresponding to a bit error rate of $\approx 10^{-6}$) for numerous w_0 settings is shown in Fig. 2(b). Unless otherwise specified, the default values for other considered parameters include $h_l = 0.4665$, $\gamma = 0.5$ A/W, $\sigma_n^2 = 10^{-14}\ \mathrm{A}^2$, $\sigma_{\mathrm{pe}} = 30$ cm, $D = 40$ mm, $w_0 = 0.05$ m, $L = 7.5$ km, and $\lambda = 1550$ nm. From Fig. 2(a), it is evident that a higher $\langle C\rangle$ can be achieved with PCBs of smaller l_c, particularly

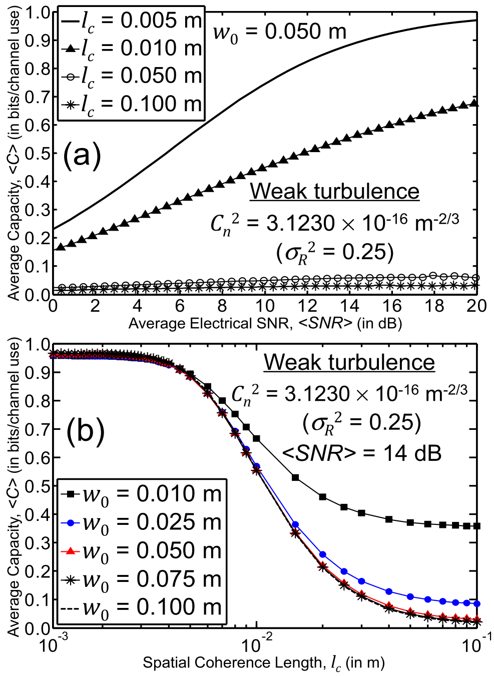

Fig. 2. (Color online) Average capacity in terms of (a) the average electrical SNR for different spatial coherence lengths and (b) the spatial coherence length for a variety of beam width settings, at $\langle \mathrm{SNR}\rangle = 14$ dB. The weak turbulence case is considered.

at larger SNR values, as compared to the laser beams of greater spatial coherence with $l_c = \{0.05, 0.10\}$ m, where $\langle C\rangle < 0.1$ bits/channel use, even with an increased SNR. We notice from Fig. 2(b) that for $\langle \mathrm{SNR}\rangle = 14$ dB, $\langle C\rangle$ exhibits a variation of >0.5 bits/channel use between the l_c extremes for all considered w_0 values. In general, Fig. 2(b) shows that w_0 does not have a significant impact on $\langle C\rangle$ for $l_c \leq 0.005$ m, but it causes performance degradation by a factor of more than 2 for the coherent case of $l_c = 0.10$ m.

Figure 3 illustrates the corresponding results for the strong turbulence case with $C_n^2 = 1.1244 \times 10^{-14}\ \mathrm{m}^{-2/3}$ ($\sigma_R^2 = 36.00$), which exhibits vastly contrasting behavior compared to the weak turbulence scenario. It is noted in Fig. 3(a) that a large l_c is preferred, but near-ideal $\langle C\rangle$ can only be attained via a substantial increase in the SNR. This shows the importance of using a coherent laser source with a high transmitting power to optimize the capacity. From Fig. 3(b) (and other turbulence cases not shown here), we observe that the maximum variation in $\langle C\rangle$ between the l_c extremes decreases significantly for stronger turbulence. In addition, an optimal $\langle C\rangle$ of 0.67 bits/channel use occurs for $l_c \geq 0.04$ m and $w_0 \geq 0.05$ m at an $\langle \mathrm{SNR}\rangle$ of 14 dB. The presented results justify the necessity for the joint optimization of w_0 and l_c, to determine the best achievable $\langle C\rangle$ under most turbulence conditions that are likely to occur in practice. For instance, through an exhaustive search over the discrete sets, we have determined the optimal values of $[w_0; l_c]^{\mathrm{opt}} = [0.10; 0.0012]$ and $[0.05; 0.0600]$ for the weak ($\sigma_R^2 = 0.25$) and strong ($\sigma_R^2 = 36.00$) turbulence conditions, respectively.

We develop an optimization metric to investigate the characteristics of the optimum beam divergence, with

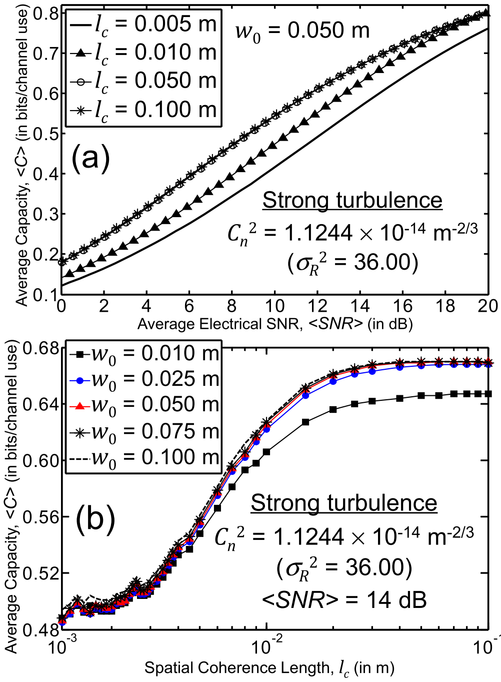

Fig. 3. (Color online) Corresponding results showing the average capacity against (a) the spatial coherence length and (b) the beam width at $\langle \mathrm{SNR} \rangle = 14$ dB, for the strong turbulence case.

respect to the changes in the turbulence strength and PEs. The beam spreading gain, $A_\zeta^{\mathrm{opt}} = w_{z_{\mathrm{eq}}}^{\mathrm{opt}} / w_{z_{\mathrm{eq}}}^{\mathrm{nom}}$, provides a feasible translation of the joint optimal transmitter beam parameters $[w_0; l_c]^{\mathrm{opt}}$ into an analogous optical beam divergence at the receiving end, with reference to the nominal coherent case of $[w_0; l_c]^{\mathrm{nom}} = [0.05; 0.10]$. Figure 4 shows the optimized $\langle C \rangle$ in terms of σ_R^2 for $\langle \mathrm{SNR} \rangle = 14$ dB. The effect of PEs, represented by the normalized jitter $2\sigma_{\mathrm{pe}}/D$, is examined. The $\langle C \rangle$ for the cases of divergent and coherent beams with $[w_0; l_c] = [0.05;\ 0.01]$ and $[0.05; 0.10]$, respectively, and the A_ζ^{opt} values are indicated for $2\sigma_{\mathrm{pe}}/D = 15.0$. It is observed that a near-ideal $\langle C \rangle$ of >0.8 bits/channel use can be achieved for $\sigma_R^2 \le 1.0$ by reducing the degree of spatial coherence, resulting in $A_\zeta^{\mathrm{opt}} > 20$. The A_ζ^{opt} must be reduced for larger σ_R^2 to maximize $\langle C \rangle$, which reaches a minimum of 1.0 for $\sigma_R^2 > 20$. For instance, A_ζ^{opt} is decreased from 21.9 to 8.4 as σ_R^2 increases from 1 to 10, thus implying that the beam divergence must be reduced by a factor of 13.5 to achieve an optimal $\langle C \rangle$ of 0.5 bits/channel use for $\sigma_R^2 = 10$. The decremental trend in A_ζ^{opt} for $2\sigma_{\mathrm{pe}}/D = 15.0$ is similarly noted for other cases. This suggests that PCBs are desirable in the weak-to-moderate turbulence regime and reveals the need of adjusting w_0 and l_c to optimize the beam divergence by a factor of A_ζ^{opt}. Coherent laser beams ($A_\zeta^{\mathrm{opt}} \approx 1.0$) operating at high transmitting power are more resilient to strong turbulence conditions. The introduction of a larger receiver aperture and joint optimization of w_0 and l_c enhances the capacity, with $\langle C \rangle > 0.8$ bits/

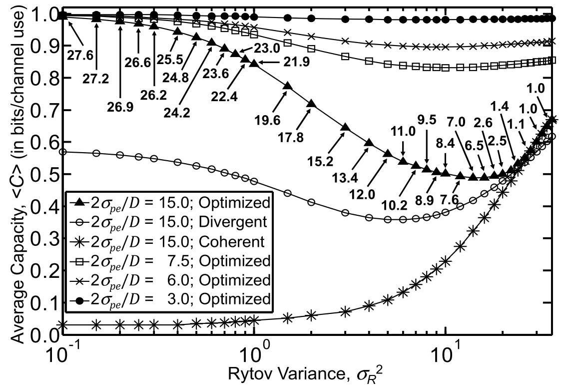

Fig. 4. Optimal average capacity in terms of the Rytov variance, for $(2\sigma_{\mathrm{pe}}/D) = \{15.0, 7.5, 6.0, 3.0\}$ at $\langle \mathrm{SNR} \rangle = 14$ dB. The cases of $[w_0; l_c] = [0.05;\ 0.01]$ (divergent) and $[0.05;\ 0.10]$ (coherent) and the values for A_ζ^{opt} are depicted for $(2\sigma_{\mathrm{pe}}/D) = 15.0$.

channel use for $2\sigma_{\mathrm{pe}}/D = \{7.5, 6.0, 3.0\}$. In principle, increasing the receiver aperture mitigates the PE loss, as indicated by the reduction in $2\sigma_{\mathrm{pe}}/D$. It also shifts the relative frequency content of the irradiance power spectrum toward lower frequencies due to aperture averaging, essentially averaging out the fastest fluctuations, and thereby reduces the scintillation.

In summary, joint beam width and coherence length optimization of partially coherent Gaussian beam has been proposed, to maximize the average capacity in FSO links over turbulence channels with PEs. The beam spreading gain has been developed as a useful metric to examine the behavior of the optimum beam divergence. Near-ideal average capacity is best achieved with an enlarged receiver aperture and the joint PCB optimization.

This research work is supported by the EU FP7 Cost Action IC 1101 on Optical Wireless Communications.

References

1. Z. Ghassemlooy, W. Popoola, and S. Rajbhandari, *Optical Wireless Communications: System and Channel Modelling with MATLAB* (Taylor & Francis, 2012).
2. A. A. Farid and S. Hranilovic, J. Lightwave Technol. **25**, 1702 (2007).
3. J. C. Ricklin and F. M. Davidson, J. Opt. Soc. Am. A **19**, 1794 (2002).
4. J. C. Ricklin and F. M. Davidson, J. Opt. Soc. Am. A **20**, 856 (2003).
5. T. J. Schulz, Opt. Lett. **30**, 1093 (2005).
6. C. Chen, H. Yang, X. Feng, and H. Wang, Opt. Lett. **34**, 419 (2009).
7. D. G. Voelz and X. Xiao, Opt. Eng. **48**, 036001 (2009).
8. D. K. Borah and D. G. Voelz, Opt. Express **18**, 20746 (2010).
9. C. Liu, Y. Yao, Y. X. Sun, J. J. Xiao, and X. H. Zhao, Opt. Lett. **35**, 3171 (2010).
10. J. Cang and X. Liu, Opt. Lett. **36**, 3335 (2011).
11. L. C. Andrews and R. L. Philips, *Laser Beam Propagation through Random Media* (SPIE, 2005).
12. E. Biglieri, J. Proakis, and S. Shamai, IEEE Trans. Inf. Theory **44**, 2619 (1998).