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Precise Gravimetric Geoid Model for Iran Based on GRACE and SRTM Data and the Least-Squares Modification of Stokes' Formula with Some Geodynamic Interpretations

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Doctoral Dissertation in Geodesy

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Abstract

Iran is one of the most complicated areas in the world from the view of rough topography, tectonic activity, large lateral density and geoidal height variations. The computation of a regional gravimetric geoid model with high accuracy in mountainous regions, especially with sparse data, is a difficult task that needs a special attention to obtain reliable results which can meet the needs of the today geodetic community.

In this research different heterogeneous data has been used, which includes gravity anomalies, the high-resolution SRTM Digital Elevation Model (DEM), recently published GRACE Global Geopotential Models (GGMs), geological maps and GPS/levelling data. The above data has been optimally combined through the least-squares modification of Stokes formula with additive corrections. Regarding the data evaluation and refinement, the cross-validation technique has been used for detection of outliers. Also, several GGMs and DEMs are evaluated with GPS/levelling data. The impact of utilizing a high resolution SRTM DEM to improve the accuracy of the geoid model has been studied. Also, a density variation model has been established, and its effect on the accuracy of the geoid was investigated. Thereafter a new height datum for Iran was established based on the corrective surface idea. Finally, it was found that there is a significant correlation between the lateral geoid slope and the tectonic activities in Iran.

We show that our hybrid gravimetric geoid model IRG04 agrees considerably better with GPS/levelling than any of the other recent local geoid models in the area. Its RMS fit with GPS/levelling is 27 cm and 3.8 ppm in the absolute and relative senses, respectively. Moreover, the relative accuracy of the IRG04 geoid model is at least 4 times better than any of the previously published global and regional geoid models in the area. Also, the RMS fit of the combined surface model (IRG04C) versus independent precise GPS/levelling is almost 4 times better compared to the original gravimetric geoid model (IRG04). These achievements clearly show the effect of the new gravity database and the SRTM data for the regional geoid determination in Iran based on the least-squares modification of Stokes' formula.

Keywords: Gravity database, least-squares modification of Stokes, geoid determination, SRTM, GRACE, GPS/levelling, density variation model, height datum, geodynamics, Iran

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Special thanks go to Dr. Jonas Ågren; many impromptu discussions with him contributed greatly to this work. His contribution in programming and software is greatly appreciated. I would like to extend my gratitude to all former and present colleagues of Divisions of Geodesy and Geoinformatics for their support. I would like to thank Drs. Tomas Egeltoft, Huaan Fan, Milan Horemuz, Mr. Erik Asenjo, Mrs. Solveig Winel and also PhD students Marek Rannala, Johan Vium Andersson, Yuriy Reshetyuk and Prosper Ulotu for their support, help and warm companionship. Special thanks to Erik Asenjo and Dr. Hans Hauska for their help with computer and network problems.

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Also, I wish to thank my friends in Stockholm, Mahnaz, Ali, Mostafa, Farhanaz, Issa and Shayesteh for helping me and my family get through the difficult times, and for all the emotional support, camaraderie, entertainment, and caring they provided.

I would like to dedicate this dissertation to my spouse *Noshin* and my daughter *Pardis*. Their support, patience and understanding have had an enormous contribution to this work. Last but not least, I would like to thank my parent for their patients and persistent encouragement over my PhD studies.

Stockholm, October 2006

Ramin Kiamehr

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Although my senses were searching the desert fatigueless; discovering nothing although finding a lot; my soul was illuminated by a thousand suns; but could never ever touch the perfection of a single atom"

Avicenna (Ibn Sina), 980–1037 A.D.
Iranian physician and philosopher

Part ONE

The Hybrid Precise Geoid Model for Iran

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Chapter 1

Introduction

"Geodesy is the discipline that deals with the measurement and representation of the earth, including its gravity field, in a three-dimensional time varying space."

(Vanícek and Krakiwsky, 1986, p.45)

1.1 Preamble

Geodesy is an interdisciplinary science, which uses space-borne and airborne remotely sensed, and ground-based measurements to study the shape and size of the Earth, the planets and their changes; to precisely determine position and velocity of points or objects at the surface or orbiting the planet, within a realized terrestrial reference system, and to apply these knowledge to a variety of scientific and engineering applications, using mathematics, physics, astronomy and computer science. Geodesy is closely related with other Earth Sciences like solid Earth physics, hydrology, atmospheric sciences, oceanography, glaciology, geophysics and geology, and thus aids our understanding of the dynamic behaviour within the solid and liquid Earth, the movements of crustal plates and the behaviour of the oceans and atmosphere. It uses some of the most advanced satellite measurements, electronic and computer technologies. Radio and visual astronomy, satellite measurement of location, space-based measurements of atmospheric and oceanic phenomena, laser and radio measurement of satellite location, use of inertial navigation and measurement systems, gravity measurement and computer modelling are all part of the work of geodesists.

One of the major tasks of geodesy is the determination of the geoid, which is defined an equipotential surface the earth gravity field which coincides on average with mean sea level. According to *C.F. Gauss*, the geoid is the "mathematical figure of the Earth", and in fact, of the gravity field. The geoid surface is more irregular than the ellipsoid of revolution often used to approximate the shape of the physical Earth, but considerably smoother than the Earth's physical surface. While the latter has excursions of the over of 8 km (Mount Everest) and −11 km (Mariana Trench), the geoid varies only about ±100 m about the reference ellipsoid of revolution.

The determination and availability of a high-resolution and more-accurate geoid model is nowadays a necessity in several geosciences, since it serves as the reference surface of other measurements and phenomena. The geoid is a basic information in different disciplines related to *Geomatics* science from navigation, mapping and surveying to construction, detecting the variations of the ocean currents, study of the interior properties of the Earth in geophysics, seismology and plate tectonics. Nowadays geoid determination is getting even more crucial, due to the development of the *Global Navigation Satellite Systems* (GNSS). These systems can offer three-dimensional positioning over the world, but without having the precise geoid model, in the field of engineering the system can be used only for two-dimensional positioning. This is due to the fact, that global positioning systems provide ellipsoidal heights, which are geometric heights, instead of orthometric heights, which have physical meanings. In order to convert the ellipsoidal height into a more useful orthometric height we need to know the geoid undulation at the station.

The determination of orthometric heights by spirit levelling is known to be a time consuming and difficult task. This is especially evident in large and topographically rough countries like Iran, where the establishment of a high resolution levelling network for all parts of the country would be impractical from the financial point of view. Moreover, levelling over areas with rough terrain, like the *Alborz* and *Zagros* Mountains in north and west or central deserts of Iran, is very tedious and time consuming. However, in order to get more accurate results using this method we need a very high resolution and accurate geoid model.

New developments and advances in the gravity field determination from satellite tracking have taken place in the last few years. Recent satellite gravity missions, such as the CHAllenging Minisatellite Payload (*CHAMP*) and Gravity Recovery and Climate Experiment (*GRACE*) have provided global scale and very accurate gravity data. However, the up-to-date, accuracy of the global geoid modelling is a few decimetres, which is not sufficient for many scientific and engineering applications. So, high resolution regional geoid models are still necessary for the most practical purposes.

1.2. Research objectives and author's contribution

Many different methods for regional geoid determination have been proposed during the years, each preferring its own set of techniques and philosophy, which makes it more difficult to judge what is the best method in a certain situation. Since 1986, several gravimetric geoid models have been published in the Iran region based on the *Remove-Compute-Restore* (RCR) (Forsberg 1990 and Sanso′ 1997), the *Helmert's scheme* (Vaníček et al. 1995) and the ellipsoidal *Bruns' formula* Ardalan and Grafarend 2004) approaches.

Based on the primary investigations, which were performed by the currently available gravimetric geoid models (Kiamehr 1997, 2001, 2003a, 2003b, 2004 and Kiamehr and Sjöberg 2005a), it was found that the standard deviation (SD) of fitting them to GPS/levelling data both in the relative and absolute senses were almost the same, or in some cases worse than using the currently available Global Geopotentioal Models (GGMs). This means that for a long period of time we have not seen much improvement in the accuracy of geoid determination in this region. However, much of this problem stems from the lacking quality and quantity of the data in the area. Result of this research clearly indicates that, regardless of the computational method, the detail assessment on choice and quality of the data plays major roles in the final result. This step is particularly important in areas with limited data, which is the case of Iran.

Recently, the quantity and quality of terrestrial gravity data slightly increased, and specially several new GGMs from the recent satellite gravimetric missions (e.g., GRACE) were released. At the same time, the new high resolution Shuttle Radar Topography Mission (SRTM) global DEM was released. The main purpose of this research is to test the potential of the Royal Institute of Technology (KTH) approach based on the Least-Squares Modification of Stokes (LSMS) formula (Sjöberg, 1984, 1991, 2003c and 2003d) as an optimum way for combination and determination of a new high-resolution geoid model for Iran. This method was successfully applied in the determination of several regional geoid models in Zambia (Nsombo 1996), Ethiopia (Hunegnaw 2001), Sweden (Nahavandchi 1998 and Ågren et al. 2006a, 2006b) and the Baltic countries (Ellmann 2004), which all, except Ethiopia, mostly have flat to moderate topography. However, the determination of a geoid model in Iran is the first experience of using the KTH approach in one of the most crucial areas in the world as viewed from the aspects of topography, geoid, density variation and geodynamics. On the other hand, since 2004, many theoretical and numerical aspects of this method have been investigated and completed, which have been tested as a complete package in this research.

Through the research, different theoretical and practical aspects in geodesy and geodynamics were studied. The most highlighted subjects are:

- The development and use of the cross-validation technique for detecting outliers and the evaluation of gridding methods of gravity data,
- Evaluation of different DEMs and studying the effect of using high resolution DEMs in accurate geoid modelling,
- The evaluation of different GGMs,
- The establishment of a density variation model for Iran and studying its effect in geoid modelling,
- Establishment of a new height datum for Iran based on the corrective surface idea and
- Studying the impact of the geoid model on tectonic activity in Iran.

The results of this research can be used as a procedure for gathering, evaluating and combining different data for the determination of gravimetric geoid models in developing countries (specially in mountainous areas), where limited ground gravity data is available. The dissertation consists of two parts: Part One (review part), which includes 10 chapters, and Part Two as the publication part, including in 9 papers.

1.2.1. Review of Part One

Chapter 1 starts with the introduction and explaining the general scope of the thesis. In *Chapters 2 and 3* we review the mathematical aspects of the KTH approach to determine the geoid based on the LSMS formula, including its additive correction terms. The Stokes method and the mathematical model for the determination of the regional gravimetric geoid are fully described in this chapter. In *Chapter 4* we explain the procedure for the creation and evaluation of the basic data, which will be used in the geoid determination, including the new generation and refinement of the gravity database, the choice of the best GGM and DEM models and GPS/levelling data. *Chapter 5* continues with a brief explanation of choosing the best modification parameters for the construction of the new Iranian geoid model (IRG04). Also, an approach for evaluating the effect of the lateral topographical mass density variation on the geoid is proposed in *Chapter 6.* The procedure for the establishment of a new height datum for Iran by combining the gravimetric and GPS/levelling geoid models is explained in *Chapter 7.*

In *Chapter 8* we evaluate and compare the accuracy of the newly released geoid models and the latest geoid models in the study area versus the GPS/levelling data in the absolute and relative senses. The internal accuracy of the geoid model was estimated by means of the expected global mean square error, whereas the GPS-levelling data is applied for an external evaluation of the accuracy of the computed geoid models.

During this research (2003) a large earthquake disaster happened in the south-east of Iran (*Bam* city), which for a while focused this author to work on anther major task of geodesy, namely *geodynamics*. *Chapter 9* includes some of geodynamical experiences of the author about the application of the geoid in studying the correlation between different geo-spatial data and earthquakes. The impact of the geoid model was studied for clarifying the tectonic boundaries of Iran. Also, we did a pilot research project about the application of the 3D deformation analysis in a GPS network in the south part of Sweden (Skåne area). Results of this research, including a brief review of the method and final results are presented in the second part of this chapter.

Finally, the study results are concluded in *Chapter 10,* which contains a general summary of all topics, a discussion of the most important results and some recommendations for future research.

1.2.2. Review of Part two

All of the research contributions described herein have already been accepted/published or submitted in peer-reviewed international scientific journals. The publications will be referred to as PAPERS A-I as follows:

PAPER A:

Kiamehr R (2005) Qualification and refinement of the gravity database based on cross-validation approach, A case study of Iran, Proc. Geomatics 2004 (84) Conferences, National Cartographic Centre of Iran, Tehran, Iran, *Revised version, submitted J. Acta Geodaetica et Geophysica*

PAPER B:

Kiamehr R and Sjöberg LE (2005a) The qualities of Iranian gravimetric geoid models versus recent gravity field missions. *J. Studia Geophysica et Geodaetica, 49:289–304*

PAPER C:

Kiamehr R and Sjöberg LE (2005b) Effect of the SRTM global DEM on the determination of a high-resolution geoid model: a case study in Iran, *J. Geodesy, 79(9):540-551*

PAPER D:

Kiamehr R (2006a) A strategy for determining the regional geoid in developing countries by combining limited ground data with satellite-based global geopotential and topographical models: A case study of Iran*, J. Geodesy , 79(10,11): 602–612*

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Kiamehr R (2006c) A new height datum for Iran based on combination of the Gravimetric and GPS/levelling geoid models, *in press Dynamic Planet 2005 IAG Symposia Series Springer Verlag, Revised version accepted and in press, Acta Geodaetica et Geophysica, 42(1)*

PAPER G:

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1.2.3 Other Publications and Presentations

The following articles published or presented during the PhD studying period by author which are not included in this dissertation:

1. Kiamehr R (2003a) Comparison Relative Accuracy of EGM96 and Iranian Gravimetric Geoid, Proc. *6Th International conference of Civil Eng*., Isfahan University of Technology, Iran, *537-544*.
2. Kiamehr R (2003b) The estimation of relative accuracy of GPS/levelling in Iran (In Persian), *Journal of Sepehr*, *11(42):13-15*.
3. Kiamehr R (2003c) Multi Object Optimization of the Geodetic Network, Proc. *Annual Geomatics 82 Conferences* (CD-ROM), Iranian National Cartographic Centre, Tehran, Iran.
4. Kiamehr R (2003d) Comparison of Ambiguity Resolution Using LAMBDA and KTH Method, *International Symposium on GPS/GNSS*, November 2003, Tokyo, Japan.
5. Kiamehr R (2004) The Relative Accuracies of Recent Satellite Gravimetric Models in Iran, Proc. *Geoid and Space Missions Symposium 2004 (GGSM2004)* (CD-ROM), August 30th-September 3rd, 2004, Porto, Portugal.
6. Kiamehr R (2004) Success Rate Determination of GPS Ambiguity Resolution Using Syntactic Data, *Position Location and Navigation Symposium (PLANS), California, USA*.
7. Ågren J, Kiamehr R and Sjöberg LE (2006) Progress in the Determination of a Gravimetric Quasigeoid Model over Sweden, *Nordic Geodetic Commission General Assembly*, May 29 to June 2, 2006, Copenhagen, Denmark.
8. Ågren J, Kiamehr R and Sjöberg LE (2006) The Swedish geoid as evaluated by the method of least-squares modification with additive, the *first International Symposium of The International Gravity Field Service (IGFS)*, August 28 – 1 September 2006, Istanbul, Turkey.

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Chapter 2

Geoid determination based on the least-squares modification of Stokes' formula

2.1 Modification of Stokes' formula

In 1849 one of the most important formulas in physical geodesy was published. This formula allows the determination of the geoidal height (N) based on gravity data. It is known as Stokes' formula or Stokes' integral after George Gabriel Stokes. It reads:

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} S(\psi)\Delta g d\sigma, \tag{2.1}$$

where R is the mean Earth's radius, ψ is the geocentric angle, Δg is the gravity anomaly, $d\sigma$ is an infinitesimal surface element of the unit sphere σ, γ is normal gravity on the reference ellipsoid and $S(\psi)$ is the Stokes function. The orthogonality properties between Legendre polynomials $P_n(\cos\psi)$ on the sphere allows us to present $S(\psi)$ as:

$$S(\psi) = \sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\cos\psi). \tag{2.2}$$

The integration expressed in equation (2.1) must be carried out over the whole earth. In practice, however, the area of integration is often limited to a spherical cap around the computation point. Molodensky et al. (1962) showed that the truncation error of the remote zone can be reduced by a modification of Stokes' formula, which combines the terrestrial gravity anomalies and the long wavelength (up to degree *M*) as contribution from a GGM.

With the satellites came the first Global Geopotential Model (GGM) and the possibility to generate global geoid models. By combining the information from the GGMs with Stokes' integration over local gravity data, local geoid models may be estimated (e.g. Rapp and Rummel 1975). The GGM gives the long-wavelength information about the geoid and the geopotential, while the local gravity data gives the short-wavelength information. Due to the use of local instead of global gravity data coverage, Stokes' kernel will be truncated at the outer range of the data coverage. This leads to truncation errors due to the loss of gravity information from the neglected outer zones. These errors may either be ignored or reduced by the modification of Stokes' kernel.

As mentioned above, the minimization of the truncation errors was first suggested by Molodensky et al. (1962) by applying a modification to Stokes' kernel. This idea has been further investigated and the modifications suggested can be divided into two categories: deterministic and stochastic types of modification. The deterministic approach intends to minimize the truncation error due to the neglected zones by removing the low degree terms (i.e. the long-wavelength part) of Stokes' kernel and high-pass filtering the gravity anomalies (Wong and Gore 1969). The major strategies have been suggested by the de Witte (1967); Wong and Gore (1969); Meissl (1971) in addition to Molodensky et al. (1962). Combinations and variants of these strategies have been suggested by e.g. Heck and Grüninger (1987) and Featherstone et al. (1998). The stochastic approach applies additional information about the potential coefficients and the gravity anomaly errors in combination with least squares modification of Stokes' kernel to minimize the expected global Mean Square Error (MSE) (e.g. Sjöberg 1984 and 1991).

Assuming a cap σ_0 of spherical radius ψ_0 of integration around the computation point, Sjöberg (2003d) presented a simple and general modification model for Stokes' formula by defining two sets of arbitrary modification parameters (s_n and b_n) as follows (cf. Sjöberg 2003d):

$$\tilde{N} = \frac{c}{2\pi} \iint_{\sigma_0} S_L(\psi) \Delta g d\sigma + c \sum_{n=2}^{M} b_n \Delta g_n^{EGM} , \tag{2.3a}$$

where $b_n = Q_n^L + s_n$, $c = R/(2\gamma)$ and $\Delta\tilde{g}_n^{EGM}$ is the Laplace harmonics of degree n can be calculated from a GGM (Heiskanen and Moritz 1967, p. 89)

$$\Delta\tilde{g}_n^{EGM} = \frac{GM}{a^2}\left(\frac{a}{r}\right)^{n+2}(n-1)\sum_{m=-n}^{n} C_{nm}Y_{nm}, \tag{2.3b}$$

where a is the equatorial radius of the reference ellipsoid, r is the geocentric radius of the computation point, GM is the adopted geocentric gravitational constant, the coefficients C_{nm} are the fully normalised spherical harmonic coefficients of the disturbing potential provided by the GGM, and Y_{nm} are the fully-normalized spherical harmonics (Heiskanen and Moritz 1967, p. 31). The modified Stokes function is expressed as

$$S_L(\psi) = S(\psi) - \sum_{n=2}^{L}\frac{2n+1}{2}s_k P_k(\cos\psi). \tag{2.4}$$

The upper limit L is arbitrary and generally not equal to M. The truncation coefficients Q_n^L can be calculated by

$$Q_n^L = Q_n - \sum_{n=2}^{\infty}\frac{2k+1}{2}s_k e_{nk}, \tag{2.5}$$

where the Molodensky's truncation coefficients Q_n can be expressed

$$Q_n = \int_{\psi_0}^{\pi} S(\psi)P_n(\cos\psi)\sin\psi d\psi, \tag{2.6}$$

and e_{nk} are the so-called Paul's coefficients (Paul 1973), being functions of ψ_0. By utilizing the error estimates of the data (terrestrial gravity anomalies Δg and the spherical harmonics Δg_n^{EGM}), and the some approximations (both theoretical and computational), we arrive at an estimate of the geoid height that we call the approximate geoidal height.

Due to the presence of errors in the estimation of GGM coefficients and terrestrial gravity data, it is possible to rewrite the estimator of Eq. (2.3a) in the following spectral form (cf. Sjöberg 2003d, Eq. 2.7)

$$\tilde{N} = c\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - Q_n^L - s_n^*\right)\left(\Delta g_n + \varepsilon_n^T\right) + c\sum_{n=2}^{M}\left(Q_n^L + s_n^*\right)\left(\Delta g_n + \varepsilon_n^S\right), \tag{2.7}$$

where ε_n^T and ε_n^s are the spectral errors of the terrestrial and GGM derived gravity anomalies, respectively. The modification parameters are

$$s_n^* = \begin{cases} s_n & \text{if} \quad 2 \le n \le L \\ 0 & \text{otherwise} \end{cases}. \tag{2.8}$$

The main objective of the modification procedure is to minimize effects of the errors in the estimation of the geoid. The modification methods proposed by Sjöberg (1984, 1991, 2003b and 2003c) allow for minimization of the truncation errors, the influence of erroneous gravity data, geopotential coefficients and combination of different data sources in the least-squares sense and at the same time in an optimum way.

The terrestrial gravity observations distributed in Iran are non-homogeneous and often affected by systematic errors. When the recent GRACE model with very high accuracy in the low to medium degrees is used, it becomes important to use a kernel modification that effectively filters out the long-wavelength errors from the gravity anomalies. For this reason, we need a proper weighting scheme for the gravity data as an *a priori* or empirical stochastic model. Of course, the true errors for the gravity data are not well-known. Here we discuss this problem shortly, as the error degree variances for GGMs and (especially GRACE only satellite models) will be estimated quite accurately, but the most crucial problem here is that the gravity anomaly error degree variances are not known. However, the least-squares method is rather insensitive to the choice of weights, as is well known from other areas of geodesy.

According to the numerical investigation performed by Ågren (2004a), the insensitivity to the weighting is illustrated by using different optimistic and pessimistic prior weighting values. Furthermore, even though the results are a bit worse, compared to the situation when the correct a priori errors are utilized, the results are still better than for the other modification methods explained above (Ågren 2004a). These numerical investigations clearly show that the least-squares method is a good alternative even though the a priori error degree variances for the gravity data are not exactly known. Some recent successful results of applications of these methods can be found in Hunegnaw (2001), Nahavandchi and Sjöberg (2001), Ellmann (2001) and Ågren (2004a).

Based on the spectral form of the "true" geoidal undulation N (Heiskanen and Moritz 1967, p. 97),

$$N = c\sum_{n=2}^{\infty}\frac{2\Delta g_n}{n-1}, \tag{2.9}$$

the expected global MSE of the geoid estimator $\tilde{N}$ can be written:

$$m_{\tilde{N}}^2 = E\left\{\frac{1}{4\pi}\iint_{\sigma}\left(\tilde{N}-N\right)^2 d\sigma\right\} = c^2\sum_{n=2}^{M}\left(b_n^2 dc_n\right) + c^2\sum_{n=2}^{\infty}\left[\left(b_n^* - Q_n^L - s_n^*\right)^2 c_n + \left(\frac{2}{n-1} - Q_n^L - s_n^*\right)^2 \sigma_n^2\right], \tag{2.10}$$

where $E\{\}$ is the statistical expectation operator, c_n is the *gravity anomaly degree variances*, σ_n^2 is terrestrial gravity anomaly *error degree variances*, dc_n is the *GGM* derived gravity anomaly error degree variances and

$$b_n^* = \begin{cases} b_n & \text{if } \; 2 \le n \le L \\ 0 & \text{otherwise} \end{cases}, \quad s_n^* = \begin{cases} s_n & \text{if } \; 2 \le n \le L \\ 0 & \text{otherwise} \end{cases}. \tag{2.11}$$

The first, middle and last terms on the right-side of Eq. (2.10) represent the contributions due to errors of the geopotential model, the truncation and the influence of erroneous terrestrial data, respectively. For all the data, the errors are assumed to be random with expectations zero, so the norm of the total error can be obtained by adding their partial contributions. However, in practice, the GGM and ground gravity data are often correlated. For example, because of using surface gravity anomalies in the construction of the EGM96 model, we can assume correlations between these two kinds of data. However, if one utilizes just "satellite-only" harmonics, this correlation can be avoided. On the other hand, we know that the terrestrial data information is comprised in the higher degrees of the GGM, but most of the geoid power is in the lower degrees, so we might assume that the influence of the correlations is insignificant.

2.2. Signal and noise degree variances

The gravity anomaly signal degree variances, the terrestrial gravity anomaly error degree variances and the GGM derived gravity anomaly error degree variances can be computed as follows:

$$c_n = \frac{1}{4\pi}\iint_\sigma \Delta g_n^2 d\sigma, \qquad \sigma_n^2 = E\left\{\frac{1}{4\pi}\iint_\sigma \left(\varepsilon_n^T\right)^2 d\sigma\right\} \tag{2.12a}$$

and

$$dc_n = E\left\{\frac{1}{4\pi}\iint_\sigma \left(\varepsilon_n^S\right)^2 d\sigma\right\}, \tag{2.12b}$$

respectively. In this study we estimate the lower signal degree variance c_n by using the spherical harmonic coefficients C_{nm} and S_{nm} of the disturbing potential from a GGM models as:

$$c_n = \frac{(GM)^2}{a^4}(n-1)^2 \sum_{m=0}^{n}\left(C_{nm}^2 + S_{nm}^2\right), \tag{2.13}$$

In practice the infinite sum in Eq. (2.10) must be truncated at some upper limit of expansion, say $n_{\max} = 10800$. The higher signal degree variances can be generated synthetically. Among three well known models for estimation of the signal gravity anomaly degree variances [e.g. Kaula 1963, Tscherning and Rapp 1974 and Jekeli 1978], the Tscherning and Rapp model yields the most realistic values for the gravity anomaly truncation RMS errors and gives reasonable RMS values for geoidal heights (Ågren 2004a). So we decided to use this model to account for the highest degrees of gravity anomaly degree variance, which are defined by

$$c_n = A \frac{(n-1)}{(n-2)(n+B)} \left(\frac{R_B}{R} \right)^{n+2}, \qquad (2.14)$$

with the parameters chosen to *R*=6371 km, *A*=225 mGal^2, *B*=4 and $R - R_B = 3.5\,\text{km}$. However, this model is valid just for the gravity field uncorrected for any topographic effects. The *error (noise) anomaly degree variance* of the erroneous potential coefficients with standard errors $d_{C_{nm}}$ and d_{Snm} (c_n) derived from the GGMs.

$$dc_n = \frac{(GM)^2}{a^4} (n-1)^2 \sum_{m=0}^{n} \left(d_{C_{nm}}^2 + d_{Snm}^2 \right). \qquad (2.15)$$

For estimating of the error degree variances for the terrestrial gravity anomalies, two different error degree variance models will be used to represent gravity anomaly errors, namely the uncorrelated and the reciprocal distance models. For the latter an isotropic error degree covariance function $C(\psi)$ is presented in the closed form (Sjöberg 1986):

$$C(\psi) = c_1 \left[\frac{1-\mu}{\sqrt{1-2\mu\cos\psi + \mu^2}} - (1-\mu) - (1-\mu)\mu\cos\psi \right], \qquad (2.16)$$

and the degree variances σ_n^2 for the *reciprocal distance* type function is given by

$$\sigma_n^2 = c_1(1-\mu)\mu^n,\ 0<\mu<1. \tag{2.17}$$

where c_1 and μ are constants. From the closed form Eq. (2.16) the parameters c_1 and μ can be computed from a knowledge of the of the variance $C(0)$ and correlation length $\psi_{1/2}$. In this research Ridder's method (Press et al. 1992) was used to find μ from $\psi_{1/2}$. Then c_1 is computed as $c_1 = \sigma^2/\mu^2$, which follows from Eq. (2.16). Both c_1 and μ are depending on the situation of the study area and the data. The value $\psi_{1/2} = 0.1^\circ$ gives a reasonable result with the Iran data which it is matching with the previous studies by (Nahavandchi 1998, Ellmann 2004 and Ågren 2004a). By choosing $\psi = 0$ in Eq. (2.16) we get:

$$\sigma^2 = C(0) = c_1\mu^2 \Rightarrow C(\psi_0) = 0.5c_1\mu^2, \tag{2.18}$$

with the solution for μ = 0.99899012912 can be found iteratively (Ellmann 2004).

In the second error model it is assumed that the observation noise is uncorrelated, which is approximately modelled by band-limited white noise with constant degree-order variances, $\sigma_{nm,\Delta g} = \dfrac{\sigma_{n,\Delta g}^2}{(2n+1)}$ (Rummel 1997 and Jekeli and Rapp 1980). If it is considered that the number of potential coefficients between degree 2 and the Nyquist degree M_N is equal to $(M_N+1)^2-4$, the degree variances can be derived as:

$$\sigma_{nm,\Delta g} = \frac{\sigma^2}{(M_N+1)^2-4}(2n+1). \tag{2.19}$$

Numerical results of this research show that the second model gives the minimum expected mean square error and fitting versus the GPS/levelling data. (See Chapter 7 for more details).

2.3. Determination of the least-squares modification parameters

To obtain the Least-Squares Modification (LSM) parameters, Eq. (2.10) is differentiated with respect to s_n, i.e., $\partial m_{\tilde{N}}^2 / \partial s_n$. The resulting expression is then equated to zero, and the modification parameters s_n are thus solved in the least-squares sense from the linear system of equations (Sjöberg 2003d):

$$\sum_{r=2}^{L} a_{kr} .s_r = h_k \,,\; k = 2,3,...,L, \tag{2.21}$$

where a_{kr} and h_k are modification coefficients, which can be expressed via Q_n ,e_{nk} ,c_n, dc_n and σ_n^2 by:

$$\begin{aligned} a_{kr} &= \left(a_k^2 + dc_k^*\right)\delta_{kr} - \frac{2r+1}{2}\left(\sigma_k^2 + dc_n^*\right)e_{kr} - \frac{2k+1}{2}\left(\sigma_r^2 + dc_r^*\right)e_{rk} \\ &+ \frac{2k+1}{2}\frac{2r+1}{2}\sum_{n=2}^{\infty} e_{nk} e_{nr}\left(\sigma_n^2 + dc_n^*\right), \end{aligned} \tag{2.22}$$

and

$$h_k = \frac{2\sigma_k^2}{k-1} - Q_k(\sigma_k^2 + dc_n^*) + \frac{2k+1}{2}\sum_{n=2}^{\infty} Q_n e_{nk}(\sigma_n^2 + dc_n^*) - \frac{2}{n-1}e_{nk}\sigma_n^2), \tag{2.23}$$

where

$$dc_n^* = \begin{cases} dc_n & for\;\; 2 \le n \le M \\ c_n & for\;\; n > M \end{cases} . \tag{2.24}$$

Depending on the quality of local gravity quality, the chosen radius of integration (ψ_0) and the characteristics of the GGM, the modification parameters s_n vary. The system of equations in Eq. (2.21) is so ill-conditioned that it often cannot be solved by standard methods like Gaussian elimination. To overcome this problem, Ellmann (2004) and Ågren (2004a) used the standard Singular Value Decomposition (SVD) procedure provided e.g. by Press et al. (1992). Ågren (2004a) concluded that the instability of the optimum choice of parameters for the unbiased LSM method is completely harmless, and the truncated SVD can be used to obtain a useful solution. After the numerical solution of s_n, the corresponding coefficients $b_n = Q_n^L + s_n$ are computed. It can be summarized here that the modification methods by Sjöberg (1984, 1991 and 2003d) attempt via minimization of the global MSE to reduce any error in geoid modelling. In PAPER (E) and Chapter 7, we explain the detail procedure in choosing the final LSM parameters for our application to Iranian data.

Chapter 3

Additive Corrections in the KTH Approach

In the determination of the geoid by Stokes' formula, Eq. (2.1), it is necessary that there are no masses outside the geoid and the gravity data should be reduced to sea level. However, because of the presence of topography and atmospheric masses (forbidden masses) above the geoid surface, we need to add some correction terms to fulfil these necessary conditions.

In the KTH computational scheme for geoid determination (Sjöberg 2003c), we use the surface gravity anomalies and the GGM for the determination of approximate geoidal heights ($\tilde{N}^0$). After that, all necessary corrections are added directly to $\tilde{N}^0$. In contrast, in the classical approaches, these corrections are usually applied so that in the first step the surface gravity anomalies are corrected by removing the effects of topographic and atmospheric external masses or reducing them inside the geoid as a *direct effect,* and then, after applying Stokes' integral, their effects are restored (*indirect effect*). Besides, since the gravity anomalies in Stokes' formula must refer to the geoid to satisfy the second condition, a reduction of observed gravity from the Earth surface to the geoid is necessary. This step is called *downward continuation* (DWC). In the KTH approach, all these separate effects are replaced by a *total topographic effect* (total effects of topographic and downward continuation). The computational procedure for estimation of the geoidal height $\hat{N}$ can be summarized by the following formula:

$$\hat{N} = \tilde{N}^0 + \delta N_{comb}^{Topo} + \delta N_{DWC} + \delta N_{comb}^{a} + \delta N_e \,. \tag{3.1}$$

where δN_{comb}^{Topo} is the *combined topographic correction*, which includes the sum of the direct and indirect topographical effects on the geoid heights, δN_{DWC} is the downward continuation effect, δN_{comb}^{a} is the *combined atmospheric correction*, which includes the sum of the direct and indirect atmospherical effects, and δN_e is the ellipsoidal correction for the spherical approximation of the geoid in Stokes' formula to ellipsoidal reference surface.

3.1. The Topographic Corrections

As mentioned before, the combined topographic effect is the sum of the direct and indirect effects, and it can be added directly to the approximate geoidal height values derived in Eq. (2.8) as:

$$\delta N_{comb}^{Topo} = \delta N_{dir} + \delta N_{indir} \simeq -\frac{2\pi G\rho}{\gamma} H^2 , \tag{3.2}$$

where ρ is the mean topographic mass density and H is the orthometric height. This method is independent of the selected type of topographic reduction (Sjöberg 2000 and 2001a). On the other hand, the direct topographic effect which is usually affected by a significant terrain effect, including possible lateral density variations, is cancelled in the combined topographic effect on the geoid.

This formula is very simple and computer efficient. Because of the fact that rough surface gravity anomalies are Stokes integrated in the KTH approach, some important comments must be taken into account in using the method. Since the rough surface anomalies are often sampled too sparsely, the numerical errors of the Stokes' integration (discretisation error) becomes large. These errors can be reduced by using a special interpolation technique (Ågren 2004a and Kiamehr 2005b, p. 548). First, make a smoothing topographic correction that result in reduced gravity anomalies, which are smoother than the original ones. Then, in the next step, the observations are interpolated to a denser grid, and the topographic correction is finally restored, i.e. the masses are restored. The standard planar Bouguer or Residual Terrain Model (RTM) anomaly can be used in the interpolation stage.

By using this interpolation technique on the smooth topographically reduced gravity anomalies, the combined estimator can be expected to yield more or less similar accuracy as when the remove-compute-restore estimator is used. Ågren (2004a) shows, using synthetic models, that this interpolation step reduces the discretisation errors significantly in the combined approach of Sjöberg (2003c), and that it improves the geoid determination result. Notice that a good DEM should be available with at least the same resolution as the new interpolated grid. Preferably it should be denser.

3.2. The Downward Continuation Correction

The analytical continuation of the surface gravity anomaly to the geoid is a necessary correction in the application of Stokes' formula for geoid estimation. It means that, after reduction of the topographic effect, the observed surface gravity anomalies must also be downward continued to the geoid. This correction is applied to the surface gravity anomalies in the classical approaches.

Traditionally, different methods are used for DWC, but using the inversion of Poisson's integral (Heiskanen and Moritz 1967; Martinec 1998; Press et al. 1992) is the most common one. The inversion of Poisson's integral is converging well for a low resolution gravity grid (e.g. $5' \times 5'$), but the convergence is doubtful for a denser grid (Martinec 1998). On the other hand, this method has large discretisation errors, especially in rough areas, and it is also an extremely time consuming procedure (Martinec 1998). Sjöberg (2003a) designed a new method for the DWC of the full field gravity anomalies. In this method, the DWC effect is computed directly for the geoid height rather than for the gravity anomaly. (He treats all corrections in this manner as an adaptive scheme.) The downward continuation effect in this case is given by:

$$\delta N_{DWC} = \frac{c}{2\pi} \iint_{\sigma 0} S_L(\psi) \left(\Delta g^* - \Delta g \right) d\sigma, \tag{3.3}$$

where Δg is the gravity anomaly at the surface computation point P and Δg^* is the corresponding quantity downward continued to the geoid. The use of the smoothed data in the Stokes operator ($\Delta g^* - \Delta g$) makes this formula particularly advantageous, and we can therefore obtain more accurate results. To sum up, the final formulas for Sjöberg's DWC method for any point of interest P based on LSM parameters can be given by (for more details, see Ågren 2004a):

$$\delta N_{DWC}(P) = \delta N_{DWC}^{(1)}(P) + \delta N_{DWC}^{L1,Far} + \delta N_{DWC}^{L2}(P)\,, \tag{3.4}$$

where

$$\delta N_{DWC}^{(1)} = H_P\left(\frac{\Delta g(P)}{\gamma} + 3\frac{N_P^0}{r_P} - \frac{1}{2\gamma}\left.\frac{\partial \Delta g}{\partial r}\right|_P H_P\right), \tag{3.5}$$

and

$$\delta N_{DWC}^{L1,Far} = c\sum_{n=2}^{M}\left(s_n^* + Q_n^L\right)\left[\left(\frac{R}{r_P}\right)^{n+2} - 1\right]\Delta g_n(P)\,, \tag{3.6}$$

and

$$\delta N_{DWC}^{L2} = \frac{c}{2\pi}\iint_{\sigma 0} S_L(\psi)\left(\left.\frac{\partial \Delta g}{\partial r}\right|_P\right)\left(H_P - H_Q\right)d\sigma_Q\,, \tag{3.7}$$

where $r_P = R + H_P$, σ_0 is a spherical cap with radius ψ centred around P and it should be the same as in modified Stokes formula, H is the orthometric height of point P and gravity gradient $\frac{\partial \Delta g}{\partial r}$ in point P can be computed based on Heiskanen and Moritz (1967, p.115):

$$\left.\frac{\partial \Delta g}{\partial r}\right|_P = \frac{R^2}{2\pi}\iint_{\sigma_0}\frac{\Delta g_Q - \Delta g_P}{\ell_0^3}d\sigma_Q - \frac{2}{R}\Delta g(P)\,, \tag{3.8}$$

where $\ell_0 = 2R\sin\frac{\psi_{PQ}}{2}$.

3.3. The Ellipsoidal Correction

Stokes' formula determines the geoidal height from the gravity anomaly on the sphere with a spherical approximation of 0.3%. However, in theory, the boundary surface for the gravity anomaly is the geoid, which can be better approximated by an ellipsoid. The spherical approximation can cause a relative error of 0.3% in the estimation of a geoid model. In precise geoid determination, this approximation should be taken into account by some correction term (ellipsoidal correction). Different authors have studied the ellipsoidal correction for the original Stokes formula, see e.g. Molodensky et al. (1962), Moritz (1980), Martinec and Grafarend (1997), Fei and Sideris (2000), Heck and Seitz (2003) and the references therein. A new integral solution for the ellipsoidal correction was published by Sjöberg (2003b). The ellipsoidal correction for the original and modified Stokes formulas is derived by Sjöberg (2003e) and Ellmann and Sjöberg (2004) in a series of spherical harmonics to the order of e^2, where e is the first eccentricity of the reference ellipsoid.

Nowadays most local geoid models are determined using a modified version of Stokes' formula, which means that the major parts of the long-wave features of the geoid are estimated from a GGM. As the GGMs can correctly be applied at sea level (approximately the ellipsoid), the remaining ellipsoidal correction for Stokes' formula is limited to the integration cap. The *ellipsoidal correction to the modified Stokes' formula* (δN_e) to order e^2 is (Sjöberg 2004):

$$\delta N_e(P) = \frac{R}{2\gamma}\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - s_n^* - Q_n^L\right)\left(\frac{a-R}{R}\Delta\tilde{g}_n^{EGM}(P) + \frac{a}{R}(\delta g_e)_n\right), \tag{3.9}$$

where $s_n^* = s_n$ if $2 \le n \le M$ and $s_n^* = 0$ otherwise. Furthermore,

$$(\delta g_e)_n = \frac{e^2}{2a}\sum_{m=-n}^{n}\left\{\left[3-(n+2)F_{nm}\right]T_{nm} - (n+1)G_{nm}T_{n-2,m} - (n+7)E_{nm}T_{n+2,m}\right\}Y_{nm}(P), \tag{3.10}$$

where T_{nm} are spherical harmonic coefficients for the disturbing potential. See Sjöberg (2004c) for the ellipsoidal coefficients E_{nm}, F_{nm} and G_{nm}. Ellmann and Sjöberg (2004) concluded that the absolute range of the ellipsoidal correction in the least-squares modification of Stokes' formula does not exceed the cm level with a cap size within a few degrees. It proves the common assumption that the ellipsoidal correction could be neglected in the modified Stokes formula and in many practical applications. However, in regions with large gravity anomalies or large integration cap, the ellipsoidal correction cannot be ignored in precise geoid modelling.

3.4. The Atmospheric Correction

As mentioned before, the presence of forbidden atmospheric masses outside the geoid surface (same as topographic masses), means that it is necessary to add an additional correction term to satisfy the boundary condition in Stokes' formula. In the computation of the atmospheric correction in the classical IAG approach, we suppose the Earth is a sphere with a spherical atmospheric ring and the topography of the Earth is completely neglected (Moritz 1992). Based on this assumption, some tabulated correction terms are usually added as a direct effect to gravity anomaly before using Stokes' formula, and the indirect effect is so small that it is usually neglected.

It was emphasised by Sjöberg (1998, 1999b, 2001a and 2006) that the application of the IAG approach using a limited cap size, and especially in the modified Stokes' formula, can cause a very significant error in the zero-order term (more than 3 m). In the KTH scheme, the *combined atmospheric effect* δN^{a}_{comb} can be approximated to order H by (Sjöberg and Nahavandchi 2000)

$$\delta N^{a}_{comb}(P) = -\frac{2\pi R\rho_0}{\gamma}\sum_{n=2}^{M}\left(\frac{2}{n-1} - s_n - Q_n^L\right)H_n(P) - \frac{2\pi R\rho_0}{\gamma}\sum_{n=M+1}^{\infty}\left(\frac{2}{n-1} - \frac{n+2}{2n+1}Q_n^L\right)H_n(P), \qquad (3.11)$$

where ρ_0 is the atmospheric density at sea level, and H_n is the Laplace harmonic of degree n for the topographic height. The elevation H of the arbitrary power v can be presented to any surface point with latitude and longitude (φ, λ) as

$$H^{v}(\varphi,\lambda) = \sum_{m=0}^{\infty}\sum_{m=-n}^{n} H^{v}_{nm} Y_{nm}(\varphi,\lambda), \qquad (3.13)$$

where H_{nm}^{v} is the normalized spherical harmonic coefficient of degree n and order m that can be determined by the spherical harmonic analysis

$$H_{nm}^{v} = \frac{1}{4\pi} \iint_{\sigma} H^{v}(\varphi,\lambda) Y_{nm}(\varphi,\lambda)\, d\sigma. \tag{3.14}$$

For more details about the properties of the DEM model which used to generate the normalized spherical harmonic coefficient H_{nm}^{v}, see Section 4.3.

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Chapter 4

Evaluation and construction of the base data

4.1. Qualification and creation of gravity anomaly database

The computation of gravimetric geoid models for the Iranian region suffers mainly from the few gravity observations available. The total area of Iran is 1,648,195 km^2, so it is simple to show that we have about one gravity point per 65 km^2. The largest gap areas are mostly located in the Zagros and Alborz mountain areas, Lout and Kavir central desert areas, Sistan & Balochestan state (in south-east of the country) and the marine areas of the Persian Gulf, Oman and Caspian seas. Fig. 4.1 shows the distribution of the gravity data in Iran. The quality of the data is questionable because they have been gathered from different organisations for different purposes and with various accuracies during long time. In order to obtain reasonable results, we started this research by collecting and investigating the gravity data available from different sources. Thus, all available gravity data has been collected for both land and marine regions and edited by a blunder-removal processing scheme to generate an optimal gravity dataset for use in geoid determination.

We propose a technique for precise cleaning of the gravity anomaly database based on the cross-validation approach. In this technique, the terrestrial gravity anomaly is compared to a GGM and the effect of topography is taken into account in this comparison. The efficiency of the cross-validation technique is illustrated both in outlier detection and in the choice of the proper gridding technique as a case study in the construction of the Iranian new gravity database. The points removed represent 4.3% of the total database, while those remaining were 25105 gravity observations. The mean value of the statistics before and after removing the outliers were 992 and 234 mGal, respectively, which shows a significant refinement in the gravity database (Table 1). (For more details, see Kiamehr 2005; PAPER A).

Table 1. Statistics of the free-air gravity anomaly before and after the blunder removal. Unit: mGal

Gravity Data	No. of data	Min.	Max.	Mean	RMS
EGM96	Model	-151.94	168.47	5.68	45.97
BGI (International Gravimetric Bureau)	9566	-163.20	234.60	9.36	50.23
NCC (National Cartographic Centre)	8949	-292.99	1220.11	94.88	280.06
Ship-borne	7610	-128.46	40.04	-53.85	32.58
All Data (Before outlier)	26125	-343.03	**992.27**	-83.61	136.65
All Data (After outlier)	25105	-163.2	**234.6**	-7.08	54.83
Database (Filled Gaps)	27172	-163.2	234.6	-6.34	54.04

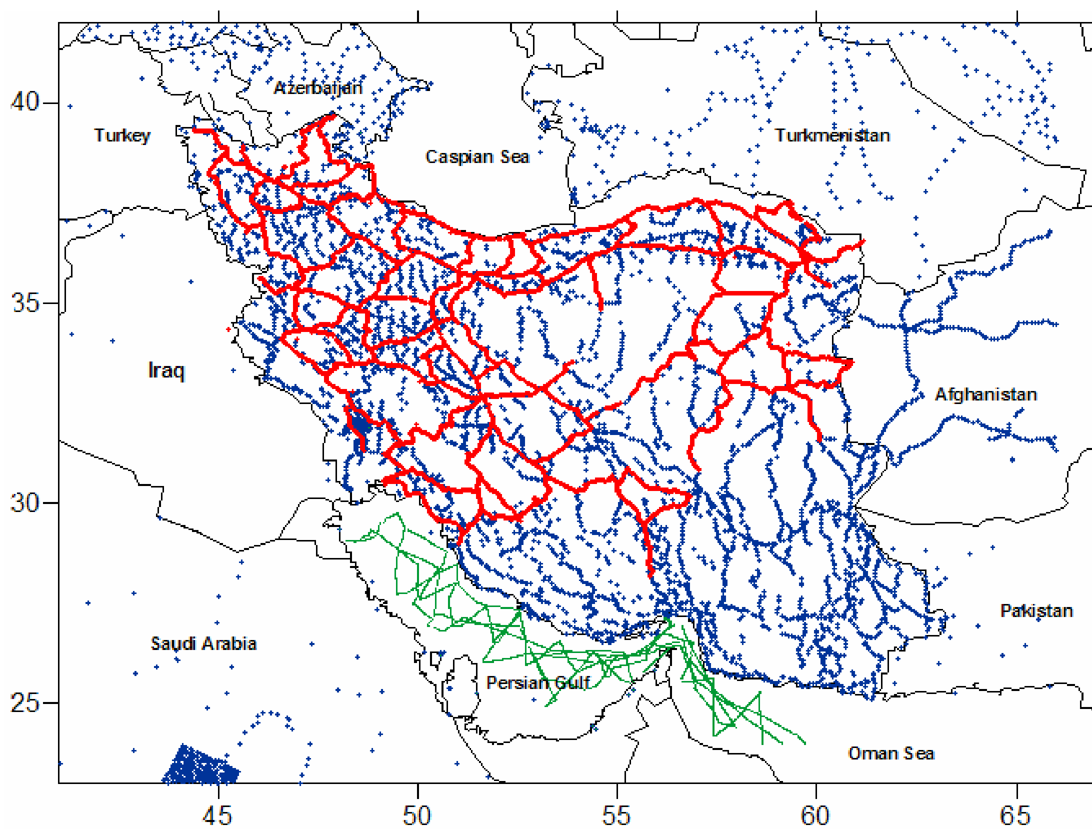

Fig. 1. Distribution of the gravity anomaly data (BGI, NCC and Ship-borne data presented in blue, red and green colours, respectively)

The geoid determination methods can be classified mainly in two categories: one uses reduced gravity data (e.g., RCR which also use topography and EGM reduced data) and other uses unreduced data (e.g., combined approach of Sjöberg 2003c). The main problem of using the unreduced gravity data in the combined model is that the free-air anomaly is known to be more sensitive to the topography. If there is rough topography in the computation area (e.g., Iran), the free-air anomalies will be rough, and due to loss of short-wavelength gravity information (so called discretisation error), the interpolation cannot always be successful. In order to overcome this problem, a special interpolation techniques was used in the gridding scheme of the gravity data (For more details, see Section 3.1 of PAPER A). The numerical investigation shows the effect of this interpolation step in reducing the discretisation errors in the KTH approach of Sjöberg (2003c). Based on the 1 km GLOBE (2003) DEM, we found a significant improvement in the order of 0.34m in the approximate geoid height between the unreduced and reduced gravity grid. (See Kiamehr and Sjöberg 2005b; PAPER C, Table 4).

The predicted $80'' \times 90''$ grid of free-air gravity anomalies is presented in Figure 2. Within the whole target area free-air anomalies vary from -182 to +352 mGal.

4.2. The GPS/levelling data

In this research GPS/levelling data were used as an external and independent tool for the estimation of the absolute and relative accuracy of the different global and local gravimetric geoid (or DEM) models. (For more details, see Ch. 7.)

The establishment and measurement of the Iranian GPS network started in August 1988 using single frequency GPS receivers. The observations and adjustment of this network was renewed and completed in recent years using dual frequency GPS receivers and data processed with Bernese software (Beutler et al., 1996). From the 260 available GPS/levelling points, 35 points belong to the first-order and the rest belong to second-order national GPS and levelling networks (Fig. 3). The mean standard deviation of the geodetic heights was estimated to 0.2 m. (Nankali, 2005)

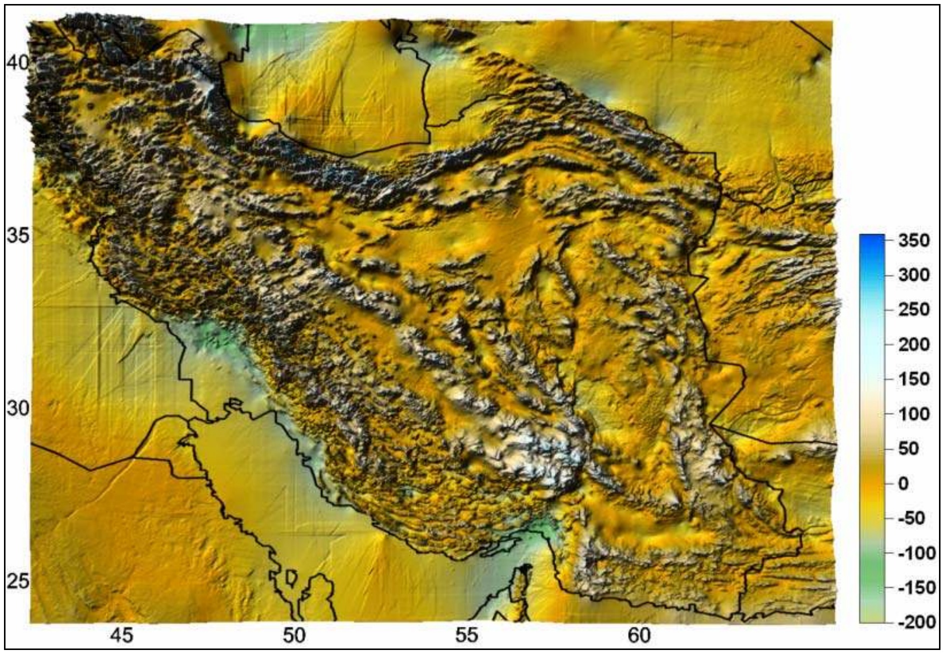

Fig. 2. The predicted $80'' \times 90''$ free-air anomaly grid. Unit: mGal

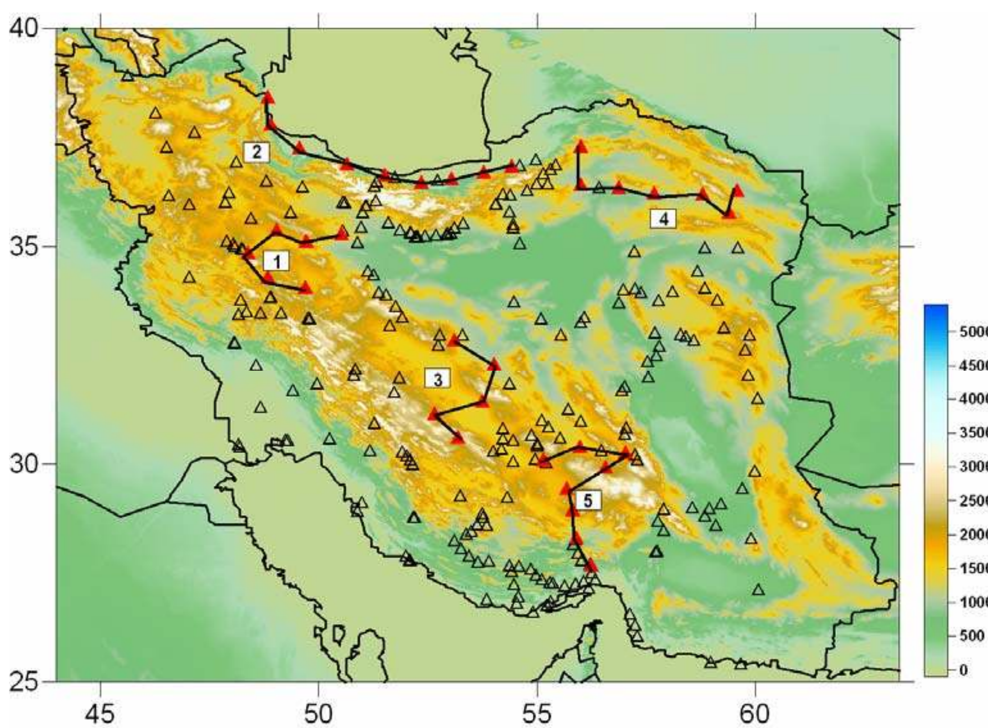

Fig. 3. Distribution of the GPS/levelling data in Iran. Red and black points indicate first and second-order GPS and levelling networks, respectively. Five traverses are chosen to studying the effect of topography in different areas. Unit: m

The estimated absolute accuracy for the first-order spirit levelled heights (35 points) is about 0.7 m. Different sources of errors e.g., neglecting the effect of the Sea Surface Topography in the 1989 adjustment of levelling networks (Hamesh 1991 and Abbolgasem 1994), the presence of systematic errors (e.g., refraction, staff settlement, neglecting the correction of gravity on orthometric heights) and some uncertainty in definition and establishment of the height datum in the adjustment of the network are reduced the absolute accuracy of the orthometric heights. However, the accuracy of the second-order levelling network should be lower than the first-order network, because it was computed based on the fixed first-order network stations. From the view of the relative accuracy, fortunately the field observation of GPS and levelling networks were performed with good observation and instrumentation standards under strict rules. According to Hamesh (1991), the average accuracy of the relative orthometric heights of the first order national levelling network is about 3 ppm.

4.3. The Digital Elevation Models

In this section, a brief overview of the current DEM of the region and its accuracy is given. Two different local DEM models have been constructed for Iran since 1997. The computation of the first Iranian DEM was conducted jointly by the Institut für Angewandte Geodäsie (IfAG), Germany, the Institute of Geophysics, Tehran University (IGTU) and the National Cartographic Centre of Iran (NCC). The height of the DEM, extracted from the 1/250000 base maps (paper version) of the country has 1 km resolution. It is clear that the accuracy of heights, in the best case estimated as 1/3 of contour interval, could be near 80 m in best case. The horizontal accuracy of this model is estimated to 125 m. This DEM is used in the computation of the current official gravimetric geoid model of Iran by the IfAG group. In 2001, NCC decided to produce a new precise national DEM based on the 1/25000 photogrammetric maps with 10 m resolution. At present, 80% of the country is covered by this DEM, but still it is under construction and evaluation, so it was not available for our research.

Because of presence of outliers and also low accuracy of the IfAG DEM, the old version of the global GTOPO model was used in the computation of some recent geoid models (e.g., Ardalan and Grafarend 2004 and Najafi 2004).

Based on the investigations by Kiamehr and Sjöberg, 2005b (PAPER C) investigations, the absolute accuracy of different DEMs in the study region were evaluated versus GPS/levelling, and also the procedure for creating a new precise DEM model for Iran was explained. We summarized that the effect of using high-resolution Shuttle Radar Topography Mission (SRTM) in gridding of gravity anomalies and terrain correction for the geoid model compared with previous DEMs is significant, and that it is important to use the new SRTM DEM for computation of the new gravimetric geoid. We found very large differences between the GLOBE and SRTM models, in the range of -750 to 550 metres. These differences cause an error in the range of -160 to 140 mGal in free-air correction and -60 to 60 mGal in the simple Bouguer plate correction. Using the adaptive formula of KTH approach for terrain correction of geoid, we got a maximum difference 3 cm between the two DEMs.

4.3.1. Iranian SRTM based DEM

The Shuttle Radar Topography Mission (SRTM) data products result from a collaborative mission by the National Aeronautics and Space Administration (NASA), the National Imagery and Mapping Agency (NIMA), the German Space Agency (DLR) and Italian Space Agency (ASI), to generate a near-global DEM of the Earth using radar interferometry.

The SRTM data flight occurred on February 11-22, 2000, and it successfully fulfilled all mission objectives. The mission collected 3-D measurements of the Earth's land surface using radar interferometry, which compares two radar images taken at slightly different locations to obtain elevation or surface-change information. The collected radar images are converted to DEMs spanning the globe between 60° North and 58° South. The "virtual Earth" will be reconstructed as a mesh of 30 m spacing, and it is accompanied for each point by a measure of the reflected energy of the radar signal, the intensity image. The SRTM is a valuable asset for many applications ranging from geodesy, geology, tectonics, hydrology, cartography, to navigation and communications.

The memorandum of understanding between NASA and NIMA for SRTM specifies that data processed at $3''$ (~100 m) for anywhere on the globe will be unrestricted, as will $1''$ (~30 m) data for the United States and its territories. Detailed documentation with technical specification of the SRTM data can be found at http://www2.jpl.nasa.gov/srtm/index.html.

The first version of SRTM data used in this research is unedited, and it contains numerous voids (areas without data), water bodies may not appear flat, and coastlines may be ill-defined. There are many holes or gaps in the old version of the SRTM data, especially in mountainous areas, also other data quality limitations are too complicated to be described here. To summarize, there is a vertical error of 16m in 90% confidence interval. This model does not include any bathymetric data.

There are several software tools for patching the gaps in the SRTM files, but no tools are available for using shoreline vectors to fix the noise on SRTM water areas. For the Iranian DEM model, the gaps in land and also marine areas were filled with ComputaMaps 500 m SRTM data. Heights of all points in the gravity data grid were extracted directly from the original SRTM (3) model with 100 m resolution. The newly release of SRTM data with significant improvements in filling mentioned gaps are now available from: http://srtm.csi.cgiar.org/index.asp.

The new Iranian DEM was computed by 15 second resolution (approximately 480 m) by use of recent 3 second (100 m) SRTM DEM data (Kiamehr and Sjöberg, 2005b). The output grid size is limited to $23^\circ < \varphi < 42^\circ$, $42^\circ < \lambda < 67^\circ$. Small gaps in land areas was patched and filled by an interpolation procedure, for the large land and marine areas information from the new version of GTOPO DEM was used. This DEM is intended to be used in interpolation of free-air anomalies and to compute topographic correction in the new geoid model of Iran. The minimum, maximum, mean and standard deviation of heights in IRD04 DEM are -84.5, 5033.1, 758.1 and 760.6 m, respectively (Fig.11). PAPAER C presents detail investigations about the procedure of the evaluation of different DEMs and their impacts on the accuracy of the geoid.

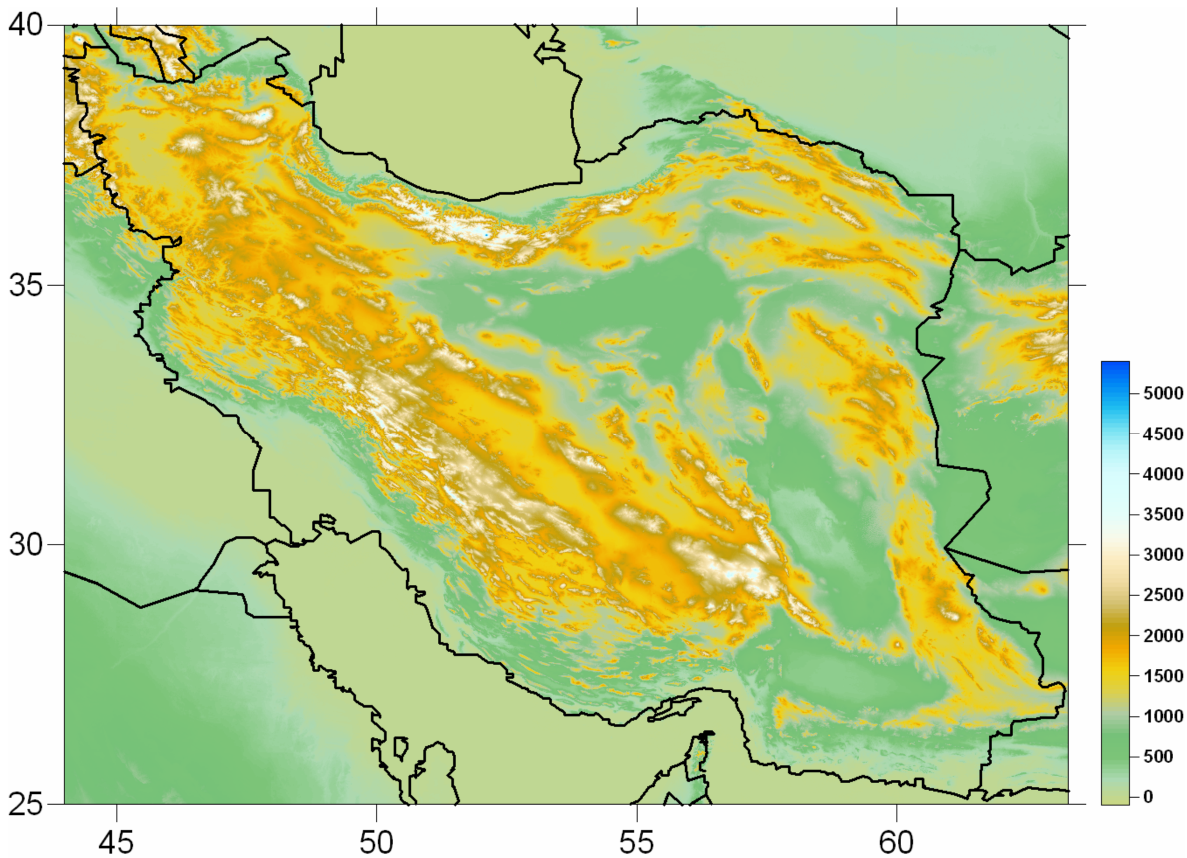

Fig. 4. The 15″ digital terrain model of Iran (IRD04) based on 3″ SRTM data.

In applying the atmospheric correction term (Eq. 3.11) in this research, a $30' \times 30'$ DEM is generated by averaging the Geophysical Exploration Technology *(GETECH)* $5' \times 5'$ DEM (GETECH 1995), using area weighting. The harmonics coefficients (H^{v}_{nm}) used here (Eq. 3.13) were provided by Fan (1998), and they were computed to degree and order 360.

4.4. The Global Geopotential Model

The new satellite gravity missions CHAMP and GRACE lead to significant improvements of our knowledge of the long wavelength part of the Earth's gravity field, and thereby of the long-wavelengths of the geoid. They provide a homogeneous and near-complete global coverage of gravity field information. Since 1990 different gravimetric local geoid models have been released for the region of Iran. During the same time several new Global Gravity Models from the recent satellite gravimetric missions CHAMP and GRACE were released. For the computation of a new gravimetric geoid model for Iran we need a new investigation on the choice of the best GGM model for the combined solution with local gravimetric data. For the validation of different global models in the absolute and relative senses, Kiamehr and Sjöberg (2005a) have used 260 GPS/ levelling data as an external tool (See PAPER B). This comparison was utilized with the new GRACE satellite-only and combined models GGM02S, GGM02C (Tapley et al. 2005) and EIGEN-02S, combined CHAMP and GRACE model EIGEN-CG01 (for more information, see http://op.gfz-potsdam.de/index_GRAM.html), GPM98C (Wenzel 1998) and EGM96 (Lemoine et al. 1998).

Among different satellite-only, combined and tailored GGMs, the study shows that the combination of the newly released GRACE model (GGM02C) with EGM96 geoid model fits the GPS/levelling data in Iran with the best absolute and relative accuracies among the GGMs. However, in practice, because of interaction between the terrestrial data and the GGM in computing the gravimetric geoid model using the least-squares modification of Stokes' formula (for more information, see PAPER B), we find that the GRACE GGM02S model gives the same results as the GGM02C and EGM96 models. Thus, we use the satellite-only GGM02S model that is more suitable for the LSMS approach. Here we give a brief presentation of the newly released GGM02 model.

4.4.1. GRACE GGM02

The GGM02S gravity model (Tapley et al. 2005) was estimated from 363 days (spanning April 2002 through December 2003) of GRACE K-band range-rate, attitude and accelerometer data. GGM02S is completely pure GRACE-only model because in its construction they do not use any Kaula constraint, other satellite information, surface gravity information and other *a priori* conditioning. The GGM02S field was estimated to degree and order 160, and the solution appears to retain the correct signal power spectrum up to about degree 120 (see Figure 6). However, it is recommended that GGM02S should not be used as is beyond approximately degree 110. (http://www.csr.utexas.edu/grace/gravity/). The GGM02C was constructed by combination of the GGM02S with terrestrial gravity information (surface gravity and mean sea surface) using the TEG4 covariance (complete to 200×200) to constrain the higher degrees to the harmonic coefficients of the EGM96. The GGM02C solution was created to degree and order 200, and retains correct signal power spectrum to this resolution, see Figure 4.4. This solution can also be smoothly extended to 360×360 by using the EGM96 coefficients to fill in above degree and order 200. Paper (B) explains detail information about the procedure of the evaluation of different GGMs in Iran.

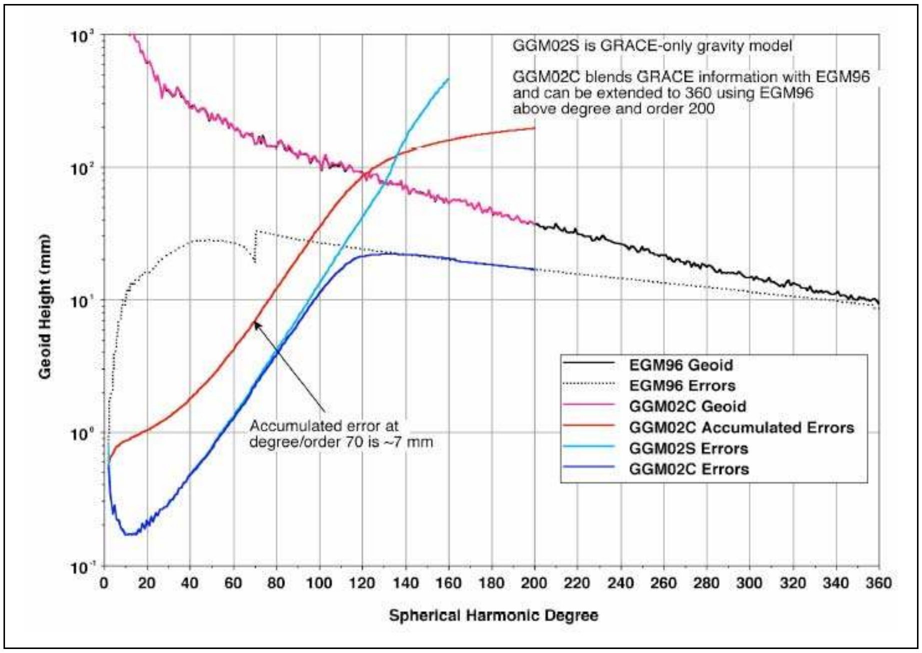

Figure 5. Estimated degree variances and degree error variances for GGM02S/C and EGM96 are shown in geoid height units. (Tapley et al. 2005).

Chapter 5

The New Gravimetric Geoid Model (IRG04)

5.1. Basic Parameters and Results

In Chapters 2 and 3 we explained the mathematical procedures for determining the geoid based on the LSMS method. The basic data including the gravity database, the GGM and DEM, were evaluated and created. These data were evaluated also through the computation of the modification parameters in the LSM approach. We tried different DEMs and GGMs with different degree and order, different integration cap sizes and also different accuracy for the gravity data to find the best parameters. These comparisons give us very good information about the properties of different sources of data and their interactions and effects in geoid models.

As we mentioned in Section 4.4, the combination of the newly released GRACE model (GGM02C) with EGM96 geoid model fits the GPS/levelling data in Iran with the best absolute and relative accuracy among the GGMs. However, as mentioned before, in practice, GRACE GGM02S model gives the same results compared combined model of GGM02C and EGM96 models. The full potential of the GGM02S model was used with the maximum degree and order of 110 in determination of the least-squares modification parameters.

So, the final geoid model was computed based on the free-air gravity anomalies in the $80'' \times 90''$ grid size, GGM02S GRACE-pure GGM and 500 m SRTM global DEM. Because of the presence of different systematic errors in gravity data and observation of data for special engineering purposes from different organisations with different accuracy, we found that there are large local correlations between data. The presence of local properties of data calls for choosing an optimum cap size for integration. We try different cap sizes (1 to 5 degrees), and found that results of the computation of geoid with a 3 degree cap works very good versus

GPS/levelling derived geoid models (See Kiamehr 2006a and 2006c, PAPER D). On the other hand, we got the best results with our pre-estimated accuracy for gravity data through the cross-validation step ($\sigma_{\Delta g}$ =10 mGal) (See PAPERS A, D and E). All additive correction terms were applied to the approximate geoidal height based on the methods, which explained in Chapter 3. Figures 6 (a-e) show the 3D view of combined topographic, DWC, total topographic, ellipsoidal correction and combined atmospheric effects on approximate geoidal heights, respectively. Figures 7 show the 3D and contour maps of the IRG04 geoid model. Notice that all corrections depend on the modification, cap size and maximum degree for the GGM being used. As mentioned before, we can see that the atmospheric and ellipsoidal corrections are small for the LSMS.

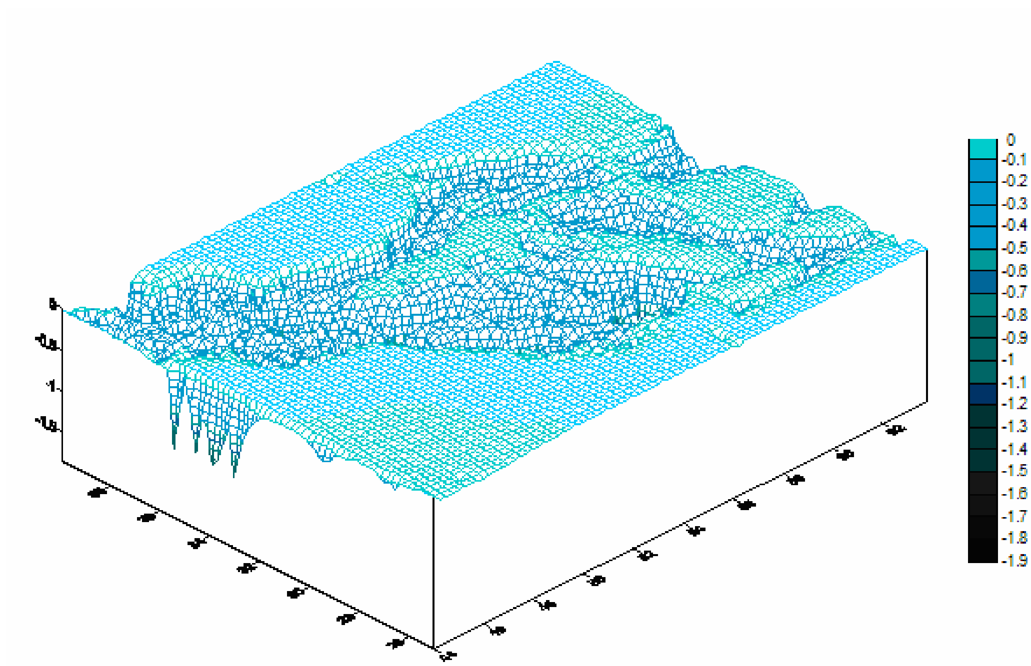

Fig. 6.a. Combined topographic effect for the geoid in Iran. Unit: m

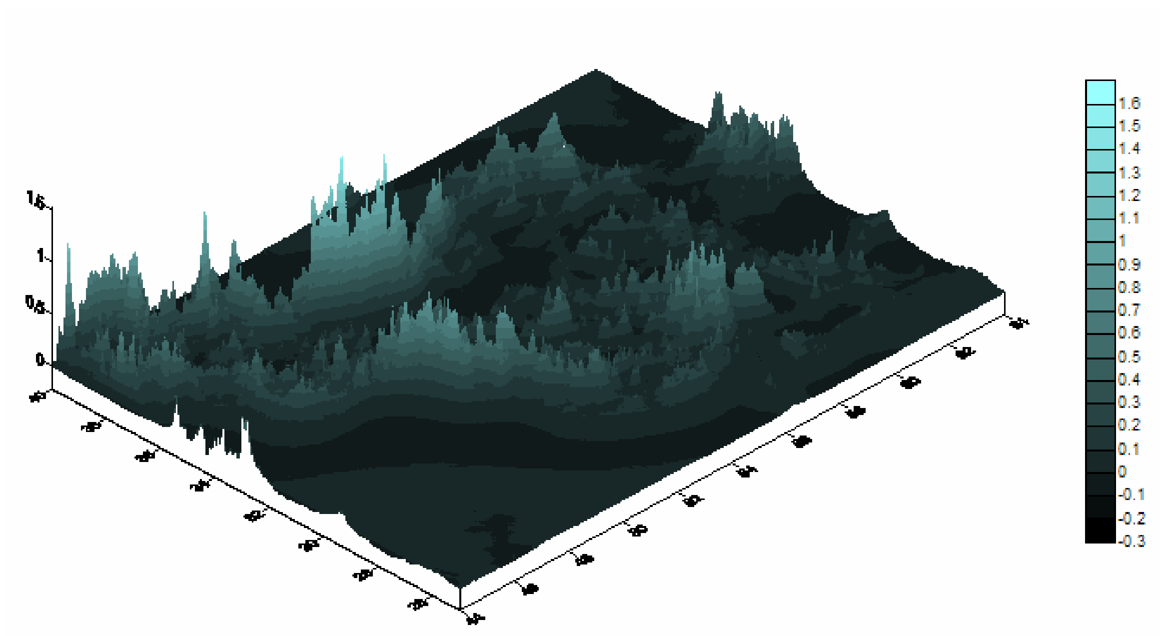

Fig. 6.b. The DWC effect on the geoid model in Iran. Unit: m

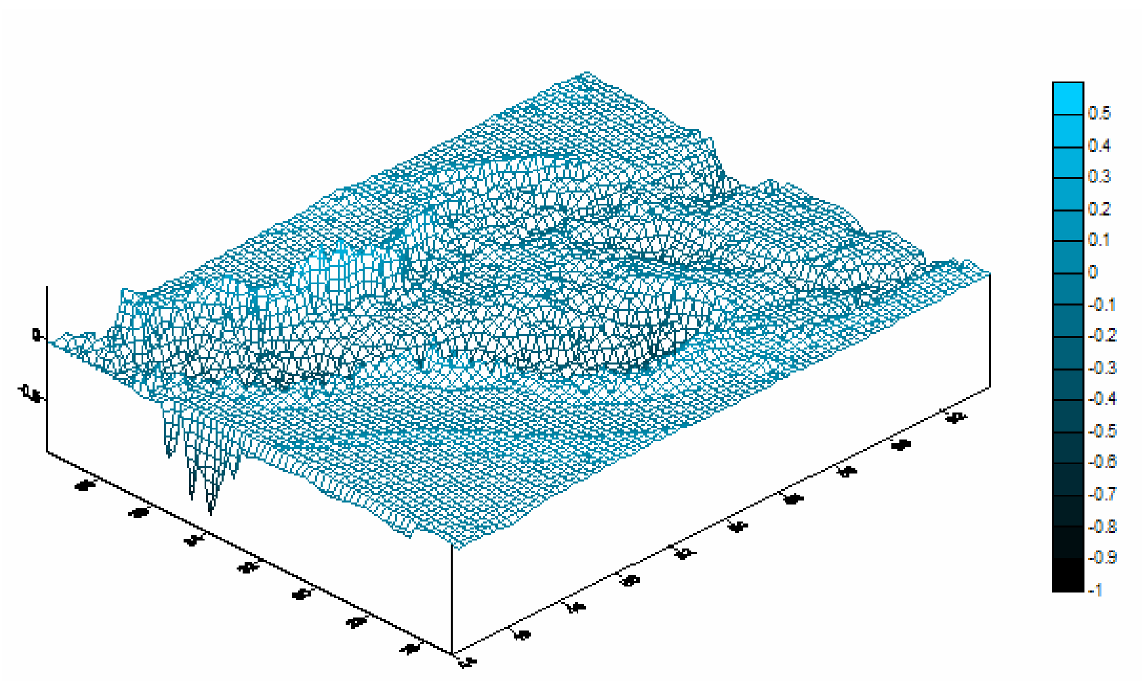

Fig. 6.c. The total topographic effect on the geoid model in Iran. Unit: m

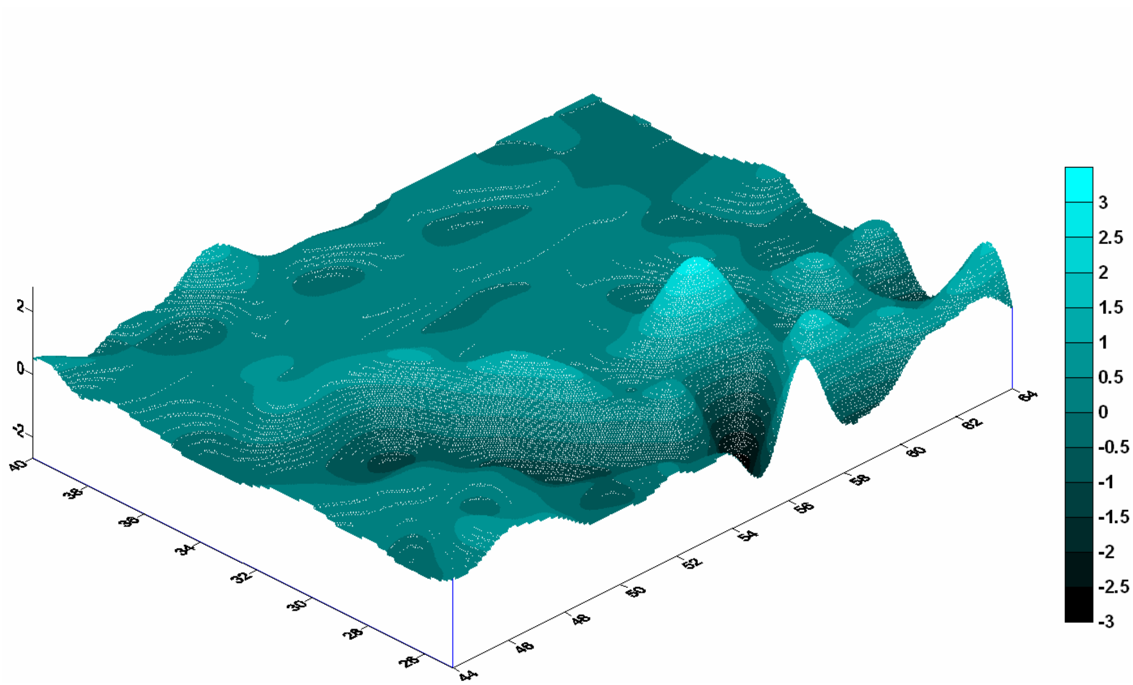

Fig. 6.d. Ellipsoidal correction on the geoid model in Iran. Unit: mm

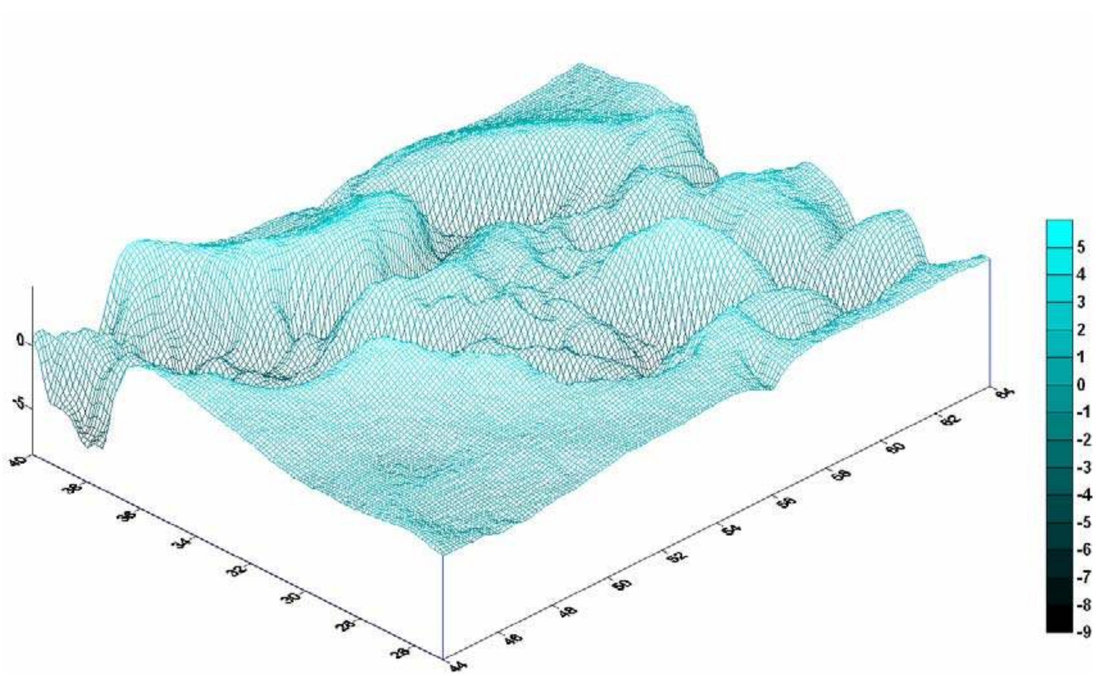

Fig. 6.e. Combined atmospheric correction on the geoid model in Iran. Unit: mm

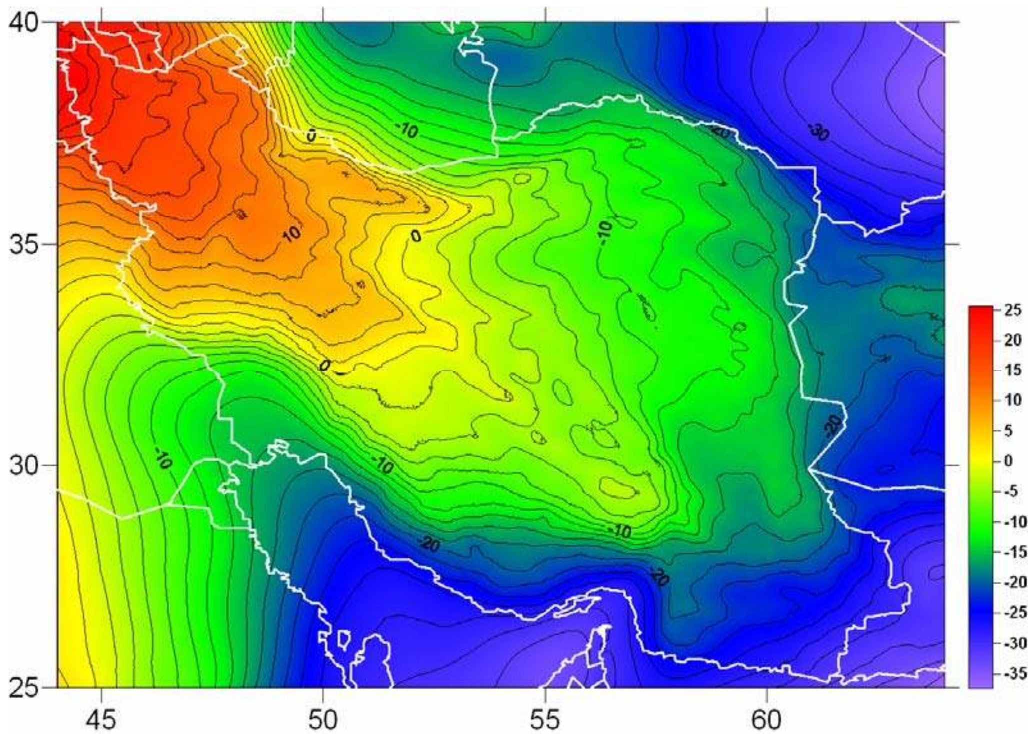

Fig. 7. The isolines of the IRG04 geoid model. Unit: m

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Chapter 6

Effect of Lateral Density Variation on the Geoid

6.1. Introduction

The existence of topography above the geoid violates the basic assumption of Stokes' formula for the determination of the geoid. Usually a constant density $2.67 g/cm^3$ is used in the determination of the geoid. This research presents the results of some preliminary experiments conducted using topographic density data in gravimetric geoid computations in Iran. There are many different reasons for starting the current research in Iran. Iran is one of the most complicated areas in the world from the view of rough topography, tectonic activity, density and geoidal height variation. The minimum, maximum, mean and standard deviation of the topographic heights in Iran are -26, 5678, 1059 and ±734 m, respectively. Also, the geoidal height varies in the range of 50 m. After the determination of the gravimetric geoid model (IRG04) (Kiamehr 2006a), we attempt to improve the accuracy of this model, specially in mountainous areas by considering the effect of lateral density variations.

In this research we utilise the very simple and practical formula for determining the effect of density variation in geoid models given by Sjöberg (2004a), that can be applied easily as an additive correction for geoid heights for any gravimetric geoid model. This method includes all density variation of the topographic and downward continuation effects. In contrast to other works, here we applied the effect of density variation in a very large area with the large density variation and also studied the density effect on the geoid based on Pratt-Hayford's isostatic model.

6.2. Topographic density models from geological maps

The Iranian Geological Survey (GSI) published the Geological Map of Iran in the scale of 1:1000000, displaying bedrock formations at or near the Earth surface (Haghipour et al. 1985). The bedrock units are grouped according to composition and geological age (see Fig. 3). Unfortunately, the digital version of this map is still not accessible for public use. The U.S. Geological Survey also published a similar geological map over Iran in scale 1:2500000 in a digital version. This map has been digitally compiled and abstracted from the original geological map of Iran, which greatly facilitates its use, by allowing a direct import into GIS software (Pollastro et al. 1999). About 541476 geometrical polygons in 59 categories are used to delimit the bedrock units over Iran. These polygons form the fundamental density units.

For generating the two-dimensional topographical mass density map using the digital geological maps in a GIS, we used legend information of the GSI geologic map, which each geological unit is assigned a range of rocks. Then we assigned a mean value of the density range as a representative density value to each geological unit by using the table of density values (Carmichael 1989 and http://www-geo.phys.ualberta.ca/~vkrav/Geoph223/Gravity-Density.htm). The resolution of a simulated density value is taken at a grid size of $80'' \times 90''$ corresponding to the gravimetric geoid model grid size. The results are displayed in Figure 8, which shows large contrasts in density variation of rocks with maximum and minimum values of 1.6 and 3.4 gm/cm^3, respectively.

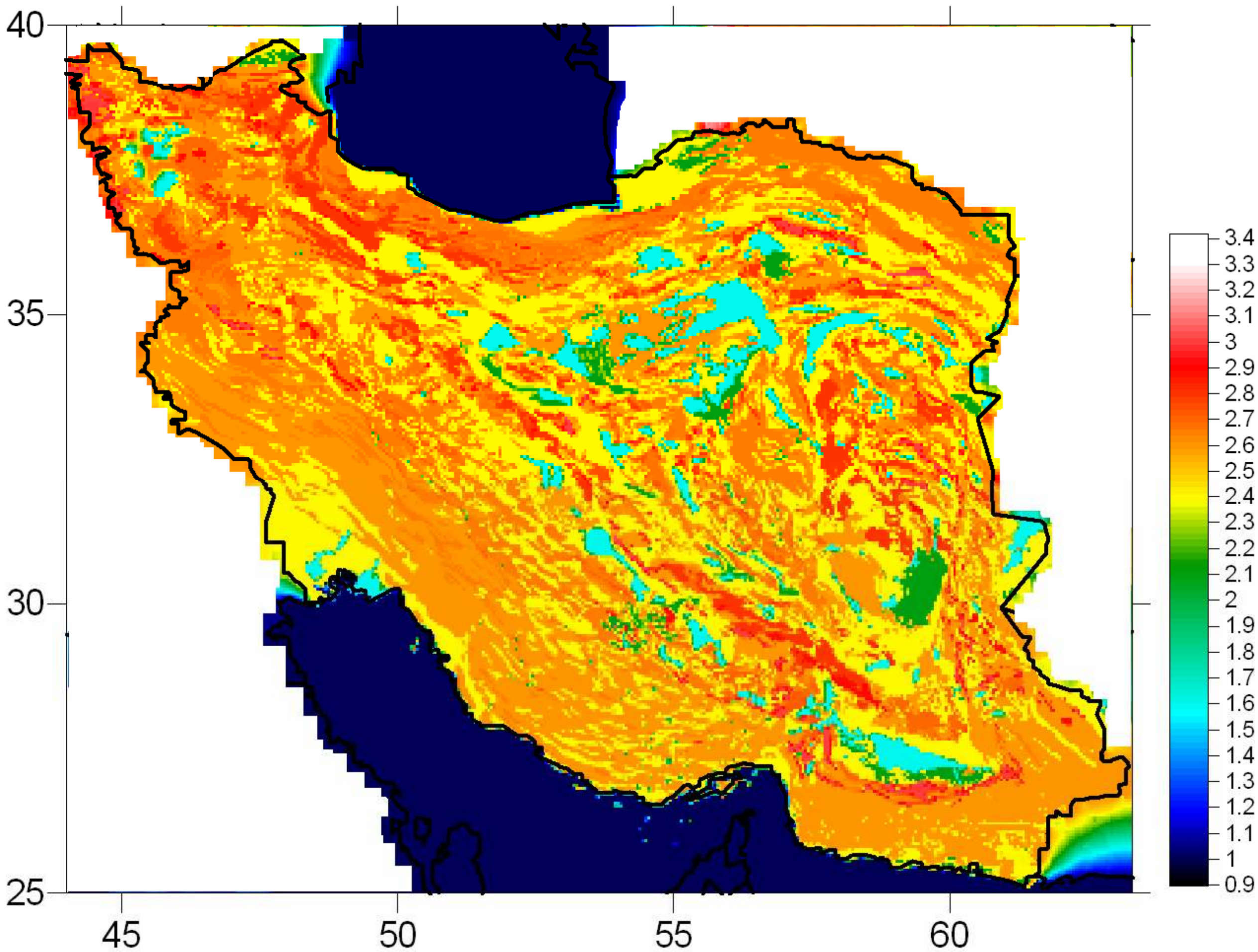

Fig. 8. The lateral density variation model of Iran. Unit: gm/cm^3

6.3. The effect on the geoid

Instead of computing separate effects of lateral density variation for direct, DWC and indirect effects, Sjöberg (2004a) showed in two independent ways that the total geoid effect due to the lateral density anomaly can be represented as a simple correction proportional to the lateral density anomaly and the elevation squared of the computation point. If the density of the topography at the computation point is

$$\rho = \rho_0 + \Delta\rho , \tag{6.1}$$

where ρ_0 is the standard density (2.67 g/cm^3) and $\Delta\rho = \Delta\rho(\theta, \lambda)$ is the lateral density anomaly with respect to the standard density, the total effect of $\Delta\rho$ on the geoid height becomes

$$\delta N_{\Delta\rho} \approx -\frac{2\pi G \Delta\rho}{\gamma_0} H^2 . \tag{6.2}$$

The topographic model used in this study, the Shuttle Radar Topography Mission (SRTM), has a resolution of 15″ (Kiamehr & Sjöberg 2005b, PAPER C). Figure 9 shows the effect of using the density variation model on the geoid. The minimum, maximum, mean and standard deviations of the lateral variation of density effect on geoid model are -0.143, 0.224, 0.004 and 0.015 m, respectively.

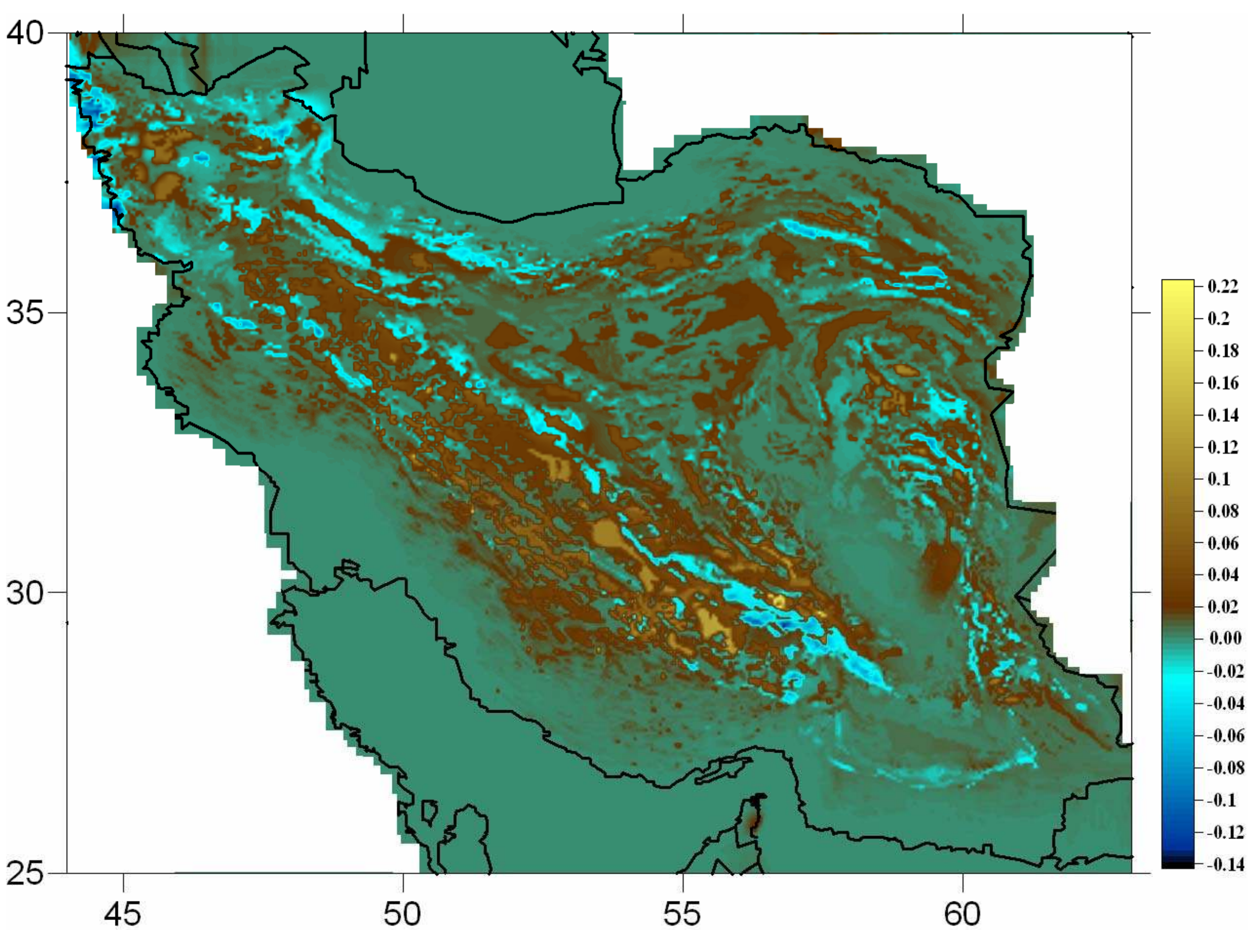

Fig. 9. Effect of the lateral density variation model on geoid. Unit. m

Also, the lateral isostatic density anomaly effect on the geoid based on the Pratt-Hayford's model (Heiskanen & Moritz 1967, p. 138) can be computed by

$$\delta N_{\Delta\rho} = \frac{2\pi G \rho_0}{\gamma_0} \frac{H^3}{D+H}, \tag{6.3}$$

where D is the depth of the crust below sea level and ρ_0 is the density for H. The minimum, maximum, mean and standard deviations of using this isostatic model in Iran are 0, 0.290, 0.008 and 0.012 m, respectively. If we consider D set to 100 km (standard value), then the minimum, maximum, mean and standard deviation of change are 0, 0.138, 0.004 and 0.005 m, respectively. Figure 10 presents the effect of using the isostatic model on geoid heights. The result indicates a significant effect on geoid based on the Pratt-Hayford's isostatic model. Our results suggest that the effect of topographical density lateral variations is significant and ought to be taken into account, specially in mountainous regions, in the determination of a precise geoid model. For more information and details, see PAPER G.

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Chapter 7

Evaluation of the Gravimetric Geoid Model

In order to show the potential of the IRG04 geoid model, we will now compare different available gravimetric geoid models in the study versus GPS/levelling data. Before this comparison, let us have a brief and quick overview of the available geoid models. Most of these models were published recently with different well-know approaches.

7.1. Overview of current Iranian Gravimetric Geoid models

Since 1986 different local gravimetric geoid models have been computed for Iran using various methods. The computation of the first (and currently only official) Iranian geoid model was conducted jointly by the Institut für Angewandte Geodäsie (IfAG) Germany, the Institute of Geophysics of Tehran University (IGTU) and the National Cartographic Centre of Iran (NCC) by using a regional geopotential model improvement approach (Weber and Zomorrodian 1988). The method was based on the GPM2 GGM (Wenzel 1985).

In a second stage, the 1 km resolution DEM (Hamesh 1992), which was extracted from scanned 1/250000 scale maps, including the OSU89B geopotential model (Rapp and Pavlis 1990) and International Gravimetric Bureau (BGI) gravity database, were utilized for determination of the geoid. The method of *remove-compute-restore* (RCR) had been used for the transformation of gravity anomalies (Δg) to geoidal undulations (N). The Fast Fourier Transform (FFT) approach was used to compute the integral formula (Hamesh 1992). Further results in this model showed that the fitting of the geoid model with GPS/levelling points improved when eliminating the terrestrial gravity data (Δg) from the geoid model, implying that the final IfAG geoid model was computed only based on the OSU89B and 1 km DTM data.

Another research project was conducted by the NCC and Department of Geodesy of Tehran University, resulting in another gravimetric geoid model (TUG) computed in a new manner based on the *ellipsoidal Bruns formula without applying Stokes' formula* (Ardalan and Grafarend 2004). They used the BGI database gravity data together with the recent accurate gravity database, which was observed by the NCC. For the computation of the terrain correction, a 1 km resolution the GLOBE global DEM was used. Recently, this group (Safari et al. 2005) published also another geoid model, which was computed based on idea of the gravimetric, satellite altimetry, astronomical *ellipsoidal boundary value problem.*

In yet another effort, a gravimetric geoid model was computed by the Department of Geodesy of K.N. Toosi University (KNTUG), using the *Stokes-Helmert scheme* (Vaníček et al. 1995). The result of this model was presented just for the central part of Iran ($30^\circ \le \varphi \le 34^\circ$ and $50^\circ \le \lambda \le 54^\circ$). The long-wavelength part of the model was determined by the CHAMP satellite-only model EIGEN-01S, and the short-wavelength contributions determined by the total BGI and NCC terrestrial gravity data and National Imagery and Mapping Agency (NIMA) GLOBE (2003) 1 km global DTM model (Najafi 2004). In Sect. 7.3 we will evaluate and compare the accuracies of these geoid models in the absolute and relative senses.

7.2. Internal Estimation of accuracy of IRG04

The internal accuracy of the geoidal height estimations can be investigated in form of the global mean square error of the estimators. The expected Global Mean Square Error (GMSE) of the unbiased least-squares model is derived as (Sjöberg 1991):

$$\delta \bar{N}^2 = c^2 \sum_{n=2}^{n_{\max}} \left[\left(\frac{2}{n-1} - s_n^* - Q_n^L \right)^2 \sigma_n^2 + \left(Q_n^L + s_n^* \right)^2 dc_n^* \right]. \tag{7.1}$$

For the IRG04 geoid model, with $n_{\max}$ =1080, maximum degree of modification and expansion *L*=*M*=110 (for GGM02S) and truncation radius of $\psi_0 = 3^\circ$, the global root mean square error is estimated to about 5.2 cm. However, we should add here the MSE is only a theoretical estimate for the accuracy of the geoid, which is usually too optimistic and does not match exactly with practical results. We estimated the GMSE based on the band-limited white noise and reciprocal distance degree variance models (Table 2). The table shows that the total expected RMS error based on the band-limited white noise model is more than 3 times better than the reciprocal distance degree variance model. So, we choose white-noise degree variance model to estimate the LSM parameters in this research.

Nowadays, the most reliable way to estimate the real potential of the gravimetric geoid is the comparison of its result with the externally derived geoidal height from GPS/levelling. This will be studied next.

Table 2. Expected global RMS-errors for the least squares modification method with pessimistic apriori error degree variances. The signal degree variances, M = L = 110, $\sigma = 10$ mGal and $\psi_0 = 3^\circ$.Unit: [mm]. (a). Band-limited white noise degree variance model (b). Reciprocal distance degree variance model.

(a)

Degrees	All	2-M	(L+1) - ∞ (10800)
Δg **part**	50.8	24.6	44.4
EGM part (GGM02S)	11	11	-
Bias (truncation error)	3.8	0.0	3.9
Total	**52.1**	26.9	44.6

(b)

Degrees	All	2-M	(L+1) - ∞ (10800)
Δg **part**	144.5	121.3	109.5
EGM part (GGM02S)	72	72	-
Bias (truncation error)	25	0.0	25
Total	**163.4**	121.3	109.5

7.3. External Estimation of the accuracy of geoid model

One of the most interesting and practical applications of a geoid model is levelling with GPS. Nowadays, for many applications, conventional spirit levelling is being replaced by height determination using GPS. It has been used for levelling projects, e.g., to monitor local subsidence due to water or natural gas removal, earth crustal movements and to control heights across water bodies in connection with a bridge construction.

The heights directly derived from GPS measurements are geodetic heights referred to the ellipsoid defined by WGS84. Figure 11 shows the basic relationship between geodetic height, (h), geoid height and orthometric height (H). In the first approximation, the heights are related by:

$$h \simeq H + N. \tag{7.2}$$

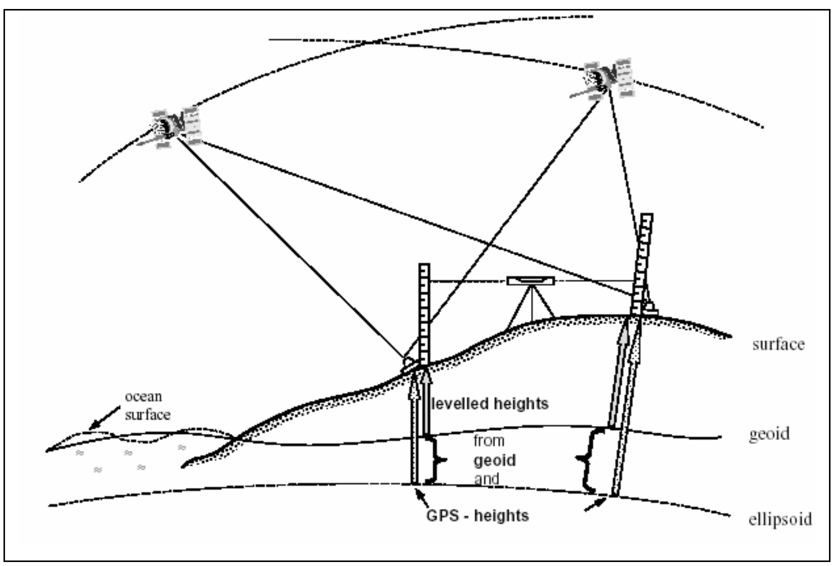


Fig. 10. Principal of levelling with GPS

The primary problem of transforming GPS-derived geodetic heights into orthometric heights thus reduces to the determination of reliable and precise geoidal undulations, which involves a combination of satellite and terrestrial gravity measurements or other data.

However, as mentioned in Chapter 4, in practice because of the presence of different random and systematic errors in the values of *h, H* and *N* [e.g., datum inconsistencies and long-wavelength systematic errors in *N*, distortions in the orthometric height datum due to an over-constrained adjustment of the levelling network, effect of various geodynamic effects (e.g., specially plate tectonic deformation in Iran), improper or non-existent terrain/ density modelling in the geoid modelling and orthometric heights and negligence of the sea surface topography (SST) at the tide gauges], the simple model in Eq. (7.2) does not work properly.

7.3.1 Estimation of accuracy in an absolute sense

In order to reduce effect of the mentioned random and systematic errors in evaluation of the geoid models, several systematic parameter models can be used in order to fit the quasi-geoid to a set of GPS levelling points through an integrated least-squares adjustment. Such a comparison is traditionally based on the following model (Kotsakis and Sideris 1999):

$$\Delta N = N_i^{GPS} - N_i = h_i - H_i - N_i = \boldsymbol{a}_i^T \boldsymbol{x} + \boldsymbol{\varepsilon}_i, \tag{7.3}$$

where $\boldsymbol{x}$ is a $n \times 1$ vector of unknown parameters, $\boldsymbol{a}_i$ is a $n \times 1$ vector of known coefficients, and ε_i denotes a residual random noise term. The parametric model $\boldsymbol{a}_i^T \boldsymbol{x}$ is supposed to describe the mentioned systematic errors and datum inconsistencies inherent in the different height data sets. Its type varies in form and complexity depending on a number of factors. The parametric models tested in this research are:

4-parameter model:

$$\boldsymbol{a}_i = \left(\cos\varphi_i \cos\lambda_i \quad \cos\varphi_i \sin\lambda_i \quad \sin\varphi_i \quad 1\right)^T \text{ and } \quad \boldsymbol{x} = \left(x_1 \quad x_2 \quad x_3 \quad x_4\right)^T. \tag{7.4}$$

5-parameter model:

$$\boldsymbol{a}_i = \left(\cos\varphi_i \cos\lambda_i \quad \cos\varphi_i \sin\lambda_i \quad \sin\varphi_i \quad 1 \quad \sin^2\varphi_i\right)^T \text{ and } \quad \boldsymbol{x} = \left(x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5\right)^T. \tag{7.5}$$

7-parameter model:

$$\boldsymbol{a}_i = \begin{pmatrix} \cos\varphi_i \cos\lambda_i \\ \cos\varphi_i \sin\lambda_i \\ \sin\varphi_i \\ \cos\varphi_i \sin\varphi_i \cos\lambda_i / W_i \\ \cos\varphi_i \sin\varphi_i \sin\lambda_i / W_i \\ \sin^2\varphi_i / W_i \\ 1 \end{pmatrix} \text{ and } \quad \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}. \tag{7.6}$$

where φ and λ are the horizontal geodetic coordinates of the network or baseline points, and

$$W_i = \left(1 - e^2 \sin^2\varphi_i\right)^{1/2}, \tag{7.7}$$

where e is the first eccentricity of the reference ellipsoid. Then we obtain the following matrix system of observation equations

$$\boldsymbol{Ax} = \Delta\boldsymbol{N} - \boldsymbol{\varepsilon}, \tag{7.8}$$

where $\boldsymbol{A}$ is the design matrix composed of one row $\boldsymbol{a}_i^T$ for each observation ΔN_i. The least squares adjustment to this equation utilizing the mean of squares of the residuals ε_i, becomes

$$\hat{\boldsymbol{x}} = \left(\boldsymbol{A}^T\boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \Delta N. \tag{7.9}$$

yielding the residuals

$$\hat{\boldsymbol{\varepsilon}} = \Delta N - \boldsymbol{A}\hat{\boldsymbol{x}} = \left[\boldsymbol{I} - \boldsymbol{A}\left(\boldsymbol{A}^T\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T\right]\Delta N. \tag{7.10}$$

The standard deviation of the adjusted values for the residuals $\hat{\varepsilon}$ are traditionally taken as the external measure of the absolute accuracy of the geoid model. It is important to mention here that the final residual values are not the exact errors of the gravimetric or GGM geoid models, because they include also some part of errors from GPS and levelling observations. According to our numerical results for these three models, the seven-parameter model gives the best fit with minimum standard deviation in all selected GGMs and gravimetric geoid models. For the evaluation of the geoid models in the absolute sense, we used all the available 260 and 35 precise GPS/levelling data separately. Table 3 (a and b) shows the results of the estimation of the absolute accuracy of IRG04 and current local gravimetric geoid models in Iran versus 260 GPS/levelling points using Eq. (7.10).

The cross-validation approach used both for detection of outliers in GPS/levelling data and evaluation of the IRG04 geoid model. Based on this investigation, from 260 GPS/levelling data, 13 points were detected as outliers. The IRG04 geoid model was evaluated again, with 247 points using the cross-validation method. Table 4 shows the results of this evaluation. By using the cross-validation approach, the overall estimation for absolute accuracy of the IRG04 model improved about 12 cm.

Table 3. Statistical analysis fitting the 260 GPS/levelling data and gravimetric geoid models from the absolute accuracy view before and after 7-parameter fitting. (The RMS value for four and five-parameter models is given in order to compare results of the fitting between different models). Unit: m

(a)

Gravimetric Geoid Models	$N_{GPS-Lev}$ - N_{KNTUG} **(Helmert Scheme) Najafi 2004** 22 points		$N_{GPS-Lev}$ - N_{IfAG} **(R-C-R Method) Hamesh et al. 1992** 260 Points	
	Before	**After**	**Before**	**After**
Min.	-11.244	-1.631	-2.46	-1.934
Max.	-4.525	1.809	2.792	3.259
Mean	**-9.559**	0.000	-0.559	0.000
RMS	1.324	**7P: 0.844**	0.801	**7P: 0.763**
		5P: 0.864		5P: 0.796
		4P: 0.911		4P: 0.775

(b)

Gravimetric Geoid Models	$N_{GPS-Lev}$ - N_{TUG} (Ellipsoidal Bruns Formula) Ardalan et al. 2004 258 points		$N_{GPS-Lev}$ - N_{IRG04} (LSMS Method) Kiamehr 2006a 260 points		$N_{GPS-Lev}$ - N_{IRG04} (Ellipsoidal boundary value problem) Safari et al. 2005 51 points	
	Before	**After**	**Before**	**After**	**Before**	**After**
Min.	-5.14	-4.261	-2.257	-1.678	-2.148	N/A
Max.	3.458	3.496	2.063	2.570	2.066	N/A
Mean	-0.517	0.000	-0.74	0.000	0.161	N/A
RMS	1.262	7P: **1.074**	0.577	7P: **0.551**	**1.068**	N/A
		5P: 1.089		5P: 0.568		
		4P: 1.123		4P: 0.571		

Table 4. Result of evaluation of the IRG04 geoid model based on the cross-validation approach, before and after seven- parameter fitting. Unit: m

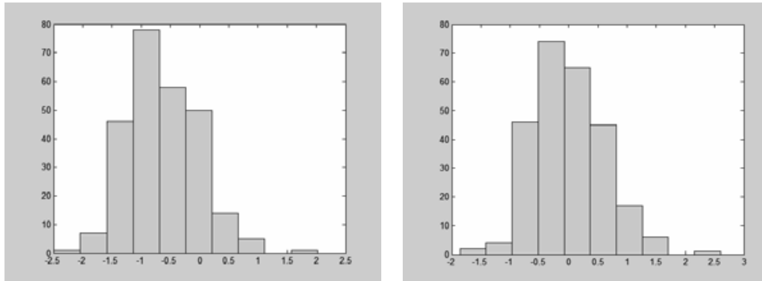
IRG04 Model (N = 247)	**Min**	**Max**	**Mean**	**RMS**
Before 7-parameter fitting	-1.891	0.489	-0.659	0.446
After 7-parameter fitting	-1.219	1.223	0.000	**0.427**

However, validation of the IRG04 geoid model based on the precise 35 GPS/levelling points, gives much better accuracy (almost 0.29 m) comparing all available GPS/levelling data (see, Table 5). As mentioned before, 225 GPS/levelling points belongs to the second-order GPS and levelling networks which have a lower accuracy comparing the 35 precise points.

Table 5. Validation of the IRG04 model versus 35 precise GPS/levelling data. Unit: m.

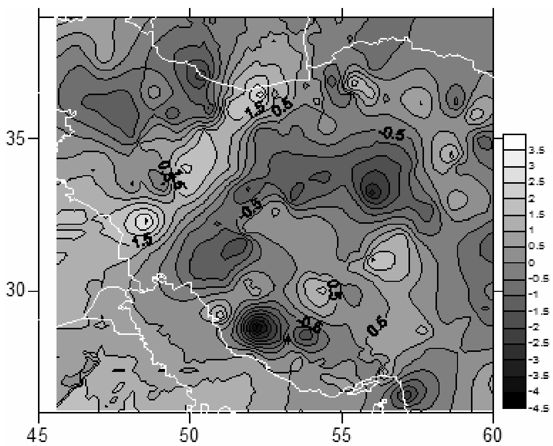
Model	**Min**	**Max**	**Mean**	**RMS**
IRG04 gravimetric geoid model	-1.284	0.223	-0.652	0.362
IRG04 after 7-parameter fitting approach	-0.518	0.924	0.000	**0.288**

Figure 11 shows the histogram of difference between the 260 GPS/levelling points and IRG04 geoid model. Figure 12 shows the discrepancy between GPS/levelling data and gravimetric geoid models in the study area.

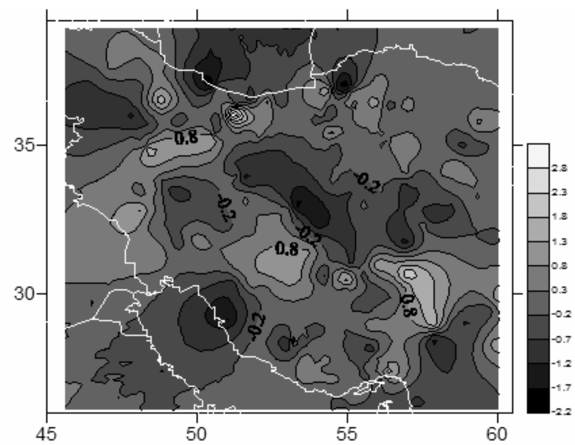


(a) **(b)**

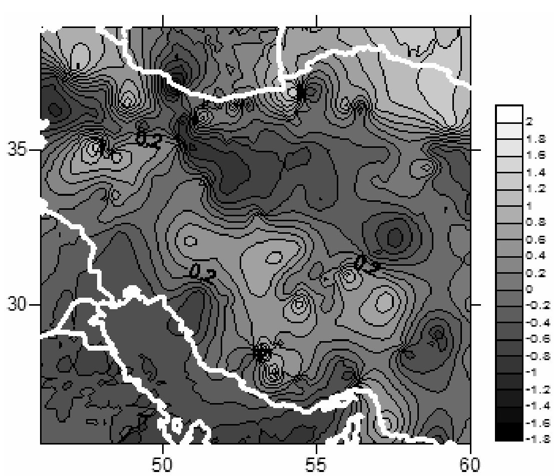
Fig. 11. Histogram of difference between the 260 GPS/levelling data and IRG04 geoid model before (a) and after 7 parameter fitting (b). Unit: m.



(a). *TUG* (***Ellipsoidal Bruns formula***)



(b). *IfAG* ***(R-C-R Method)***



(c). *IRG04* ***(LSMS formula)***

Fig. 12. Discrepancy between the GPS/levelling and three gravimetric geoid models. (Contour interval 0.5 m). Unit: m

7.3.2. Effect of lateral density variation model

In order to study the effect of using the density variation models on the IRG04 geoid model, 260 GPS/levelling points were used. The minimum, maximum, mean and standard deviations of these levelled heights are -21.45, 2551.55, 1083.68 and 651.99 m, respectively. We added either the effect of actual or isostatic models on the gravimetric geoid model as correction term (Fig. 9). Table 6 shows a summary of a statistical analysis for the evaluation of the two corrected geoid models versus GPS/levelling data.

Table 6. Effect of using the different density variation models (real and isostatic) on the IRG04 gravimetric geoid model. (Comparison is done before applying the 7 parameter fitting approach).

Geoid models	With actual density variation	With Isostatic density variation	With constant density value
Minimum	-2.257	-2.257	-2.257
Maximum	2.059	2.073	2.064
Mean	-0.751	-0.674	-0.741
RMS	0.576	0.589	0.578

Although there is a slight indication that the correction for lateral density variation works in the right direction, from Table 1 we cannot see a significant improvement by using the actual density variation model, because most of the GPS/levelling data are located in the moderate topographic areas with a maximum height of 2551 m. Also, we think that the evaluation of the effect of density models with GPS/levelling data seems not very reasonable, because basically our levelling Helmert orthometric height is not a true orthometric height. This difference is due to in the error in the estimation of the mean value of gravity along the plumb line between the surface and the geoid.

The Helmert height is based on a model of an infinite Bouguer plate with a uniform density of 2.67 g/cm^3. Variations in density and topographic relief will cause departures of Helmert heights from true orthometric heights. As a gauge on the influence of rock density variation, Heiskanen and Moritz (1967, pp.169) show a 4 mm error in Helmert height for a point at 1000 m elevation and with a constant 0.1 g/cm^3 surface density departure from 2.67 g/cm^3. Such error is proportional, so that if one assumes an average density of 2.87 g/cm^3 (e.g., diorite/gabbro combination or an alkaline basalt as found in the Rocky Mountains) distributed as a Bouguer plate with an elevation of 3000 m, then one would obtain a Helmert height error of 0.024 m. For any realistic evaluation of the effect of actual density on the gravimetric geoid model, we need some GPS/levelling points in rough areas as well as correct orthometric heights for them.

7.3.3. Estimation of accuracy in a relative sense

In order to give more precise and acceptable results, the geoid models were evaluated also in the relative sense by using the 35 precise GPS/levelling data. For this purpose, we computed the orthometric height differences using the relative geoidal undulations ΔN for selected baselines by using Eq. (7.2) as follows:

$$\Delta \mathrm{H}_{\text{GPS-GGM}} = \Delta h_{GPS} - \Delta N_{GGM}\,, \tag{7.11}$$

where Δ symbol means difference. We can easily derive differences between two different orthometric height differences, i.e. from levelling (ΔH_{Level}) and from GPS minus GGM ($\Delta \mathrm{H}_{\text{GPS-GGM}}$) using Eq. (7.11):

$$\delta\Delta \mathrm{H}_{\text{GGM-level}} = \Delta H_{GPS-GGM} - \Delta H_{Level}\,. \tag{7.12}$$

The relative differences between a GGM/gravimetric geoid model and levelling becomes in part per million (ppm):

$$\text{ppm} = \text{mean}\left|\frac{\left(\delta\Delta H_{GGM-Level}\right)_{mm}}{D_{ij_{(km)}}}\right|, \tag{7.13}$$

where D_{ij} is the length of the baseline. The average distance between the 35 GPS/levelling points is 80.65 km. Table 7 shows the results of comparison between the IfAG, TUG and IRG04 geoid models from the view of relative accuracies on five selected traverses (see Fig. 3). As the KNTUG model is available only in a limited area, we can just find two precise GPS/levelling points there, so it is not possible to make any comparison for this model.

Table 7. Statistical analysis fitting the 35 precise GPS/levelling data and gravimetric geoid models from the relative view of accuracy (by Eq. 7.13). Unit: m

Traverse	***TUG*** ***(Ellipsoidal Bruns formula)*** RMS	***IfAG*** ***(R-C-R method)*** RMS	***IRG04*** ***(LSMS formula)*** RMS
1 (West)	1.942	1.17	0.48
2 (North)	1.183	1.20	0.58
3 (Centre)	No DATA	1.33	0.48
4 (East)	1.93	0.49	0.18
5 (South)	0.407	2.00	0.14
Min.	-2.686	-3.419	-0.82
Max.	2.551	2.750	0.79
Mean	0.009	0.132	0.02
RMS	1.239	1.310	0.40
ALL Baselines (ppm)	**15.4**	**16**	**3.8**

With simple comparison between results of Tables 3 and 7, we found that the comparison of geoid models from the relative accuracy sense gives very interesting and realistic results about the real potential of gravimetric geoid models.

In order to show a better view of the improvement in accuracy of IRG04 model, we also make a comparison with the GRACE satellite-only (GGM02S) and the combined (GGM02C) models in the absolute and relative senses (Tables 8 and 9). The statistical analyse of absolute and relative accuracy for recent GRACE models before and after 7-parameter fitting procedure are shown in Tables 8 and 9.

Table 8. Statistical analysis fitting the GPS/levelling data and GRACE-GGM02 models from the absolute accuracy view, before and after applying the 7-parameter fitting procedure. Unit: m

GGM	GGM02S GRACE degree and order 160		GGM02C GRACE degree and order 200	
	Before	**After**	**Before**	**After**
Min.	-4.076	-3.671	-2.96	-2.558
Max.	2.714	3.013	1.87	2.018
Mean	-0.239	0.000	-0.243	0.000
RMS	**1.230**	**1.218**	**0.854**	**0.837**

Table 9. Statistical analysis of fitting between the 35 precise GPS/levelling data and GRACE GGM02 models from the view of relative accuracy (Eq. (8)) for GRACE GGM02 models. Unit: m

Traverse	GGM-02S degree and order 160	GGM02C degree and order 200
	RMS	RMS
1	0.451	0.500
2	2.461	1.530
3	1.860	1.477
4	0.778	0.733
5	1.348	1.223
Min.	**-3.221**	**-2.598**
Max.	**3.569**	**1.943**
Mean	**-0.01**	**0.03**
RMS	**1.540**	**1.11**
ALL (ppm)	**19**	**13.8**

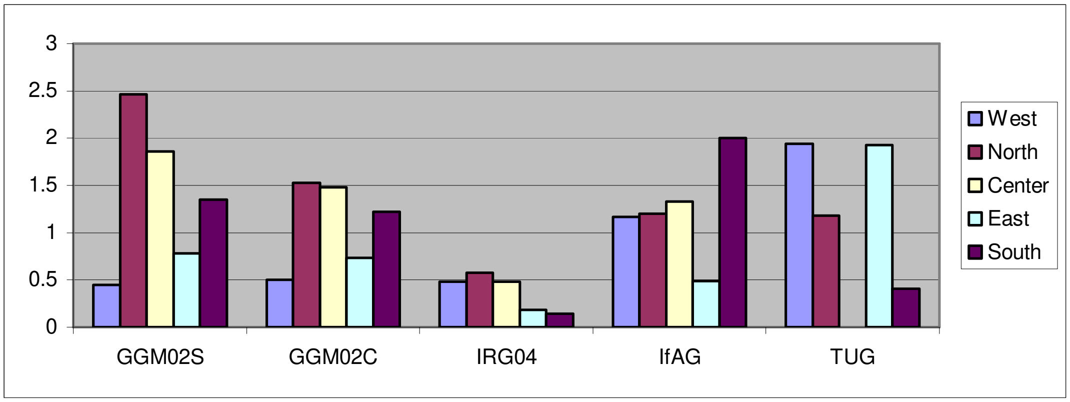


Fig. 13. Comparison of relative accuracies of fitting GPS/levelling data and GGM02 models versus local gravimetric geoid models in five selected traverses. Unit: m

It is evident that most of the progress in IRG04 is because of using the most recent GGM and DEM. However, except using some extra satellite altimetry data and EGM96 (original data) in marine and gap areas inside and outside of boarders, the quantity of the used ground data in IRG04 is almost the same as in KNTUG and TUG. We think that there are three different reasons that come into play:

a) Our method for eliminating outliers from the gravity database.

b) The interpolated denser gravity observations using the high-resolution SRTM DEM before Stokes' integration. This operation improved the RMS of the fit between the geoid model and GPS/levelling data up to 0.37 m (Kiamehr and Sjöberg 2005)

c) From the theoretical point of view, the LSM kernel matches the errors of the terrestrial gravity data, the GGM and the truncation errors in an optimum way.

Figure 14 shows the discrepancies between IRG04 and TUG. The largest differences are mostly located in the rough topographic areas in the Albourz and Zagrous mountains (north and west). We think that a large part of the differences can be attributed to the high-resolution SRTM DEM used in IRG04, which is considerably more accurate than the GLOBE DEM used in TUG. The IRG04 model fits better with GPS/levelling data in these areas, which supports this conclusion. Another factor, that might explain the differences between IRG04 and TUG, is that

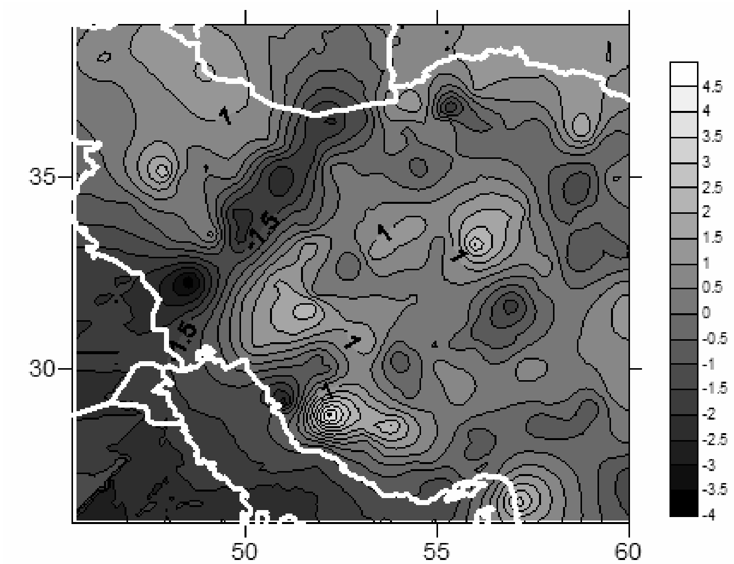


Fig. 14. Discrepancies between the IRG04 and TUG geoid models. Contour maximum and minimum are +4.5 m (brightest region) and -4 m (darkest region), respectively, and contour interval is 0.5 m.

the most recently released GRACE-only GGM is used in the former. It is also interesting to note that the GGM02C has better accuracy compared to both the TUG and RCR models (see Tables 7 and 9). In Papers D and E we present a general comparison between several geoid models using different interpolation methods, cap sizes and standard deviations of $\sigma_{\Delta g}$ and GGMs.

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Chapter 8

A New Height Datum for Iran

The statistical behaviour and modelling of the misclosure of Eq. (7.2), computed in a network of GPS/levelling benchmarks, have been the subjects of many recent studies (e.g. Kotsakis et al. 2001a and 2001b, Kotsakis and Sideris 1999 and Fotopoulos 2003). Using the corrective surface idea is one of the most interesting and practical subjects in this area. The development of corrective surfaces can be used for the optimal height transformation between ellipsoid and levelling datum surfaces for reducing the long-wavelength gravimetric geoid errors. Some studies from recent years emphasising this fact can be mentioned, like Kotsakis and Sideris (1999) and Lee and Mezera (2000).

8.1. Creation of the corrective surface

In order to define a new height datum for Iran, we attempt to combine the IRG04 high resolution gravimetric geoid model with GPS/levelling data by using a corrective surface idea. The corrective surface is constructed based on 224 GPS/levelling points and then evaluated with the 35 independent points. By applying the 7-parameter fitting approach (Eq. 7.6), the adjusted values for the residuals ($\hat{\boldsymbol{\varepsilon}}$) are estimated based on the least-square model (Eq. 7.10). Then, the continuous surface is generated from the discrete GPS/levelling data by using the prediction techniques. Different interpolation techniques were tested for the creation of the corrective surface; among them the Kriging method gives the minimum RMS and minimum spread of residuals versus the GPS/levelling data. The use of a corrective surface to the IRG04 gravimetric geoid model has been shown to significantly improve the determination of heights from GPS in Iran. The RMS of fitting the new combined model IRG04C versus GPS/levelling data is 0.09 m, which gives a more than 4 times better fit compared with the IRG04 model (see Table 10). Hence, this corrective surface should be convenient and useful in the definition of a new height datum, specifically in engineering and GPS/levelling projects. (For more information and details about the definition of the new height datum, see Kiamehr 2006c, PAPER E)

Table 10. Validation of the IRG04 and the new combined geoid model versus 35 GPS/levelling data points. Unit: m.

Model	Min	Max	Mean	RMS
IRG04 model	-1.284	0.223	-0.652	0.362
IRG04 after 7-parameter fitting	-0.518	0.924	0.000	0.288
The combined model (IRG04C)	-0.474	0.433	0.000	0.088

Chapter 9

Geodynamical Researches

As mentioned in the introduction of the thesis (Chapter 1), during this research we also make some investigations about geodynamical applications of geodesy. There are two published papers in this area in Part 2 of the thesis. In the fist paper (Kiamehr and Sjöberg 2006, PAPER G) we studied the impact of different geospatial information (specially a high resolution geoid model) in studying of tectonic activity of Iran. The second Paper (Kiamehr and Sjöberg 2005c, PAPER H) explains the analysis of some surface deformation patterns based on the 3D finite element method. In this chapter we have a brief review of the most important parts of these two articles.

9.1. Impact of the geoid model in studying tectonic activities in Iran

Geoid signals can give important information about the underlying density structure of the earth, and it can be used to locate the source depth of mass anomalies. The importance of the geoid in geophysics and geodynamics has been recognised by the studies of the correlation between the geoid and deep-Earth mass density anomalies (Bowin 1983); the constraints provided by the geoid on mantle rheology and flow (Hager, 1984); correlation between the geoid and westward drift of the geomagnetic field (Khan, 1971), geoid bulge and its correlation with land uplift and Moho depth (Sjöberg 1983 and Sjöberg et al. 1991) and correlation between the geoid and plate tectonic features and seismic tomography (Silver et al., 1988)

The Iran region is known as an extraordinary natural laboratory for the study of seismo-tectonic processes. This area is geologically and geophysically as well as geodetically one of the most studied regions on Earth. It is one of regions that are located in the line of the most recent foldings along the length of the Alp strip in Europe to the Himalayas in the north of India. Here Iran is in the place of the junction of the Saudi-Arabia, India and Eurasia plates. This junction has led to the seismicity of Iran, and, as a result, most regions and cities of Iran are exposed to frequent earthquakes.

An integrated approach, combining gravity, geoid, and topography and seismology data, allow us to have a detail study of the crustal and lithospheric structure of the region. In the current study we used the most recent information, which come from the high resolution Shuttle Radar Topography Mission (SRTM), free air gravity grid database (Kiamehr, 2005a) and the new precise combined geoid model (IRG04C), the complete earthquakes database of Iran, which was gathered by the International Institute of Earthquake Engineering and Seismology (IIEES). The seismological database has 5900 records, and it includes the general information of the earthquakes since 1902. (See Kiamehr and Sjöberg 2006d, PAPER G - Table 2).

The statistical analysis of the extracted heights of the earthquake locations from the SRTM model shows that the mean height of the seismological database points is 1091 m, but the classification of earthquakes with the magnitude over 6 says that their mean height is 1275 m. We can conclude that the flat and moderate topographic areas (like the central of Iran) have a lower risk of earthquakes compared to the mountainous areas in the north and south-west.

The gravity anomaly is a differential quantity, exhibitively small scale structure, whereas the geoid is an integrated quantity, being optimal for studying long-wavelength effects and major tectonic features. A detail study of the thematic map of the free-air anomaly and the statistical analysis of the extracted values from the grid for the seismological data (see PAPER H, Fig. 2 and Table 2) clearly shows very local and complicated structures (like topography), which are mostly related with local mass variations and shallow density contrasts below sea level, and they could therefore be useful in studying local tectonic patterns. The large positive anomalies in the Alborz and Zagros in Figure 2 are interpreted to be a result of higher-density mantle lithosphere. Combining these two observations, we conclude that the higher-density material in the uppermost mantle is the likely candidate for the cause of this anomaly.

Similar to gravity, the geoid reflects mass variations in the Earth's interior of various wavelengths, but, in contrast to gravity, the geoid primarily reflects the long and medium wavelengths. Hence, it is more useful in studying global and regional mass anomalies in the

Earth, such as deep lithosphere structures. We found interesting results from the gradient map of the geoid (geoid deflection map). It clearly shows the presence of significant correlation between the lateral geoid gradient (*NS*) and the distribution of earthquakes in Iran (see Fig. 15). Our statistical analysis shows that almost 72% of the earthquakes in Iran occurred in areas with geoid slopes over 5%. The minimum, maximum, mean and standard deviations of earthquake magnitudes are 1.7, 7.7, 4.15 and 0.84 Richter, respectively. It is also interesting that all of the 78 earthquakes with magnitude over 6 Richter are in areas with geoid slope 7.5% or more. The statistical analysis of correlation values between different parameters gives us very interesting results which can be concluded as follows (For more information see Kiamehr and Sjöberg 2006d and PAPER G):

- All correlations are significant except those between *NS* (geoid slope) and *N* (geoid height) as well as between *NS* and *R* (earthquake magnitude).
- There is a strong correlation between topographic height and each of free-air anomaly and simple Bouguer anomaly.
- There is a positive and significant correlation between the geoid slope and earthquakes depth parameters in the order of +0.46 in the Zagros area.
- There is a significant and remarkable correlation between the depth and magnitude of earthquakes in the study area ($r = 0.4$).
- The geoid slope parameter (*NS*) and free-air anomaly have the highest correlation value among different parameters with the depth of the earthquake (*D*). It could be an important sign from the earthquake study view. We think using the simple and clear map of the geoidal slope and the complicated map of the free-air anomaly are complementary for the local and regional interpretations of earthquakes and plate tectonics.

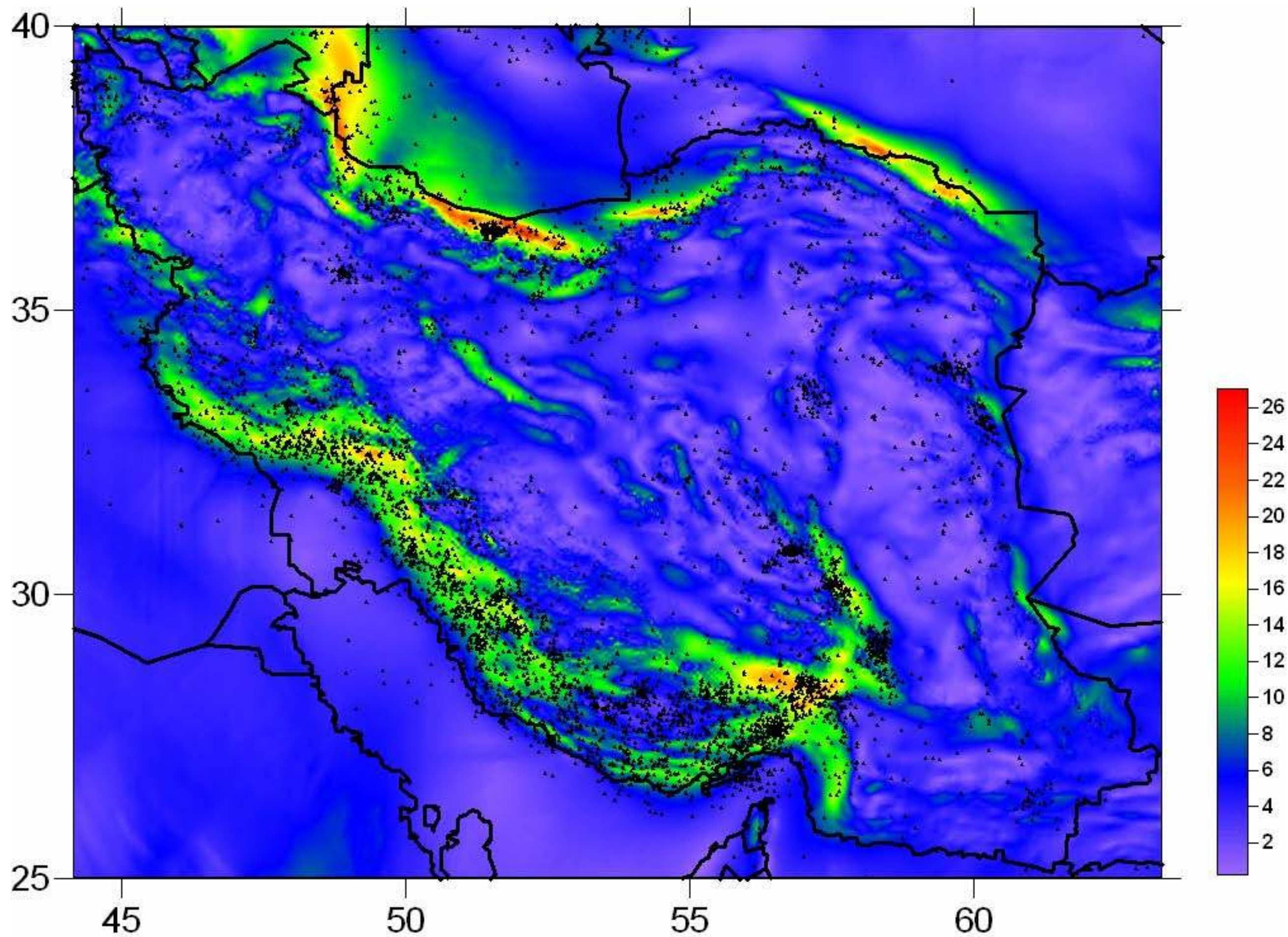

Fig. 15. Distribution of earthquakes (black dots) on the lateral geoid slope map. (Slope in percentage)

9.2. Analysis of surface deformation patterns using 3D finite element method

Typical geodetic observables are discrete functions in space and time, and, consequently, the height and displacement vectors, deduced from the geodetic data are of discrete nature. The *finite element method* (FEM) (e.g., Burnett 1987) has found a manifold of applications in geodesy and particularly in Earth deformation studies, because a geodetic network can be viewed as a typical example for a set of irregularly shaped finite elements in two or three dimensions. The domain of the problem is divided into smaller regions or sub domains, called *elements.* Adjacent elements touch each other without overlapping, and there are no gaps between the elements. The shapes of the elements are intentionally made as simple as possible, such as triangles and quadrilaterals in 2-dimensional domains, and tetrahedral and pentahedra in 3-dimensions. It yields what is known as a *Delaunay triangulation,* one in which the triangles formed are as near equilateral as possible for the given positions of irregularly spaced nodes. The analysis of geodetic observations is typically performed with models involving coordinates, which refer to more-or-less arbitrary reference frames. In this way the determination of the displacement vector can not show the real physical behaviour of the body, because it is completely dependent to the chosen reference frame.

Since deformation refers to the change of a shape, independent of position (Grafarend and Schaffrin 1976), it is only reasonable to study it by properly defined parameters, which are independent of frame definitions. Here, we first have a quick view to the principal 3D FEM. Then, we will use FEM for determination of deformation parameters in the area of Skåne in the south of the Sweden. The detail investigation and information about the principal and the theoretical aspects of this method can be found in Kiamehr and Sjöberg 2005c, PAPER I.

Here we summarize the practical procedure for the determination of the deformation parameters in a 3D view (Dermanis and Grafarend 1983). For modelling the three dimensions FEM we use 4 vertices pentahedra element, say $\boldsymbol{P}_\alpha, \boldsymbol{P}_\beta, \boldsymbol{P}_\gamma, \boldsymbol{P}_\delta$. The linear field of coordinate changes from X to X' is given by the transformation

$$\boldsymbol{X}' = \boldsymbol{J}\boldsymbol{X} + \boldsymbol{G} = \left[\boldsymbol{x}'_\alpha \boldsymbol{x}'_\beta \boldsymbol{x}'_\gamma \boldsymbol{x}'_\delta\right]_{3\times 4} = \boldsymbol{J}_{3\times 3}\left[\boldsymbol{x}_\alpha \boldsymbol{x}_\beta \boldsymbol{x}_\gamma \boldsymbol{x}_\delta\right]_{3\times 4} + \left[\boldsymbol{g} \quad \boldsymbol{g} \quad \boldsymbol{g} \quad \boldsymbol{g}\right]_{3\times 4}, \tag{9.1}$$

where $\boldsymbol{J}$ and $\boldsymbol{g}$ are the Jacobian matrix and the translation vector, respectively. Based on this model, the three-dimensional Jacobian matrix $\boldsymbol{J}$ can be computed directly from the coordinates using the solution given by

$$\boldsymbol{J} = \begin{pmatrix} x'^1_\beta - x'^1_\alpha & x'^1_\gamma - x'^1_\alpha & x'^1_\delta - x'^1_\alpha \\ x'^2_\beta - x'^2_\alpha & x'^2_\gamma - x'^2_\alpha & x'^2_\delta - x'^2_\alpha \\ x'^3_\beta - x'^3_\alpha & x'^3_\gamma - x'^3_\alpha & x'^3_\delta - x'^3_\alpha \end{pmatrix} \begin{pmatrix} x^1_\beta - x^1_\alpha & x^1_\gamma - x^1_\alpha & x^1_\delta - x^1_\alpha \\ x^2_\beta - x^2_\alpha & x^2_\gamma - x^2_\alpha & x^2_\delta - x^2_\alpha \\ x^3_\beta - x^3_\alpha & x^3_\gamma - x^3_\alpha & x^3_\delta - x^3_\alpha \end{pmatrix}^{-1}. \tag{9.2}$$

The strain matrix $\boldsymbol{S}$ can easily be computed by

$$S = \frac{1}{2}\left(J^t J - I\right). \tag{9.3}$$

If v_i are the unit *eigenvectors* of $\boldsymbol{S}$ corresponding to the *eigenvalues* e_i, the strain matrix can be diagonalised as:

$$S = V \Lambda V^t \Leftrightarrow V^t S V = \Lambda = Diag\left(e_1, e_2, e_3\right), \tag{9.4}$$

where $\boldsymbol{V}$ is the orthogonal matrix with columns being the *eigenvectors* v_i, and Λ is the diagonal matrix with diagonal elements being the ordered *eigenvalues* e_i of S. Also, the planar *dilatation* (Δ) and *maximum shear strain* (γ) components can be defined for each of the three principal planes perpendicular to the principal directions v_1, v_2, v_3, respectively:

$$\begin{aligned} &\Delta_{12} = e_1 + e_2, \quad \gamma_{12} = e_1 - e_2, \\ &\Delta_{23} = e_2 + e_3, \quad \gamma_{23} = e_2 - e_3, \\ &\Delta_{13} = e_1 + e_3, \quad \gamma_{13} = e_1 - e_3. \end{aligned} \tag{9.5}$$

The total dilatation is given by:

$$\Delta = \frac{1}{3}\left(\Delta_{12} + \Delta_{23} + \Delta_{13}\right), \tag{9.6}$$

and the direction of each principal axis is defined by the corresponding vector $v_i = \begin{pmatrix} v_i^1 & v_i^2 & v_i^3 \end{pmatrix}$, or by two appropriate direction angles, e.g. "longitude" and "latitude":

$$\Lambda_i = \arctan\left(\frac{v_i^2}{v_i^1}\right) \text{ and , } \Phi_i = \arctan\left(\frac{v_i^1}{\sqrt{\left(v_i^1\right)^2 + \left(v_i^2\right)^2}}\right). \tag{9.7}$$

9.2.1. Skåne GPS network

Since 1993 a large and complete GPS network with more than 40 stations with continuous daily observations established in the Swedish part of the Baltic shield for studying postglacial rebound and tectonic activities under the BIFROST project (Scherneck et al. 1998 2000 and 2001). This network is useful specially for studying and analysis of the general activities of major faults and geodynamics phenomena in the large scale. Nowadays, using local geodetic networks with Electronic Distance Measurement (EDM) or GPS observations is so common for studying local geodynamic activities (e.g. landslide monitoring) and analysis of deformation for large man made structures (e.g. dam, bridges and nuclear buildings). Instead of using national geodynamic networks (e.g. BIFROST) which usually have continuous observations, for local projects, based on the field conditions and limitation and specially because of economic reasons, we frequently prefer to have repeated observations in separate campaigns.

Pan et al. (2001) processed the GPS data of the Skåne campaigns in 1996 and 1998. The Skåne network is a local network, which was basically established for monitoring possible plate tectonic activity in this area. The major guidelines of how they dealt with the data are as follows. The data were processed session by session, and normal equations for all the sessions were stored. The results were then combined to obtain final solutions for each campaign of 1992, 1996 and 1999. In processing the GPS carrier phase data, the coordinates of one station usually have to be fixed (as known) in the WGS84 or the International Terrestrial Reference Frame (ITRF) coordinate system. The fiducial station Stavershult was selected as the fixed station in the ITRF96 system. (For more detail see, PAPER I)

The results of the research shows that the areas with maximum shear strain and dilation (regions G, B and A) are the most active areas and are located exactly in the active fault zones and their intersections (see Figure 5, PAPER I). However, further observations in a denser network as well as integration with geological and geophysical data are needed to fully explore the recent crustal motions over the Tornquist zone.

Chapter 10

Conclusions and Recommendations

The computation of a regional gravimetric geoid model with proper accuracy in a developing country with sparse and limited data is a difficult task, which needs a special notice to produce good results. In this research we try to investigate the procedure for gathering, evaluating and combining different data for the determination of a gravimetric geoid model for Iran. The least-squares modification of Stokes' method by the KTH approach was used for combining different heterogeneous data in an optimum way.

The first and most important step for geoid determination is to evaluate and choose the best available data in the region. The basic data which we used in this research are gravity anomaly data (from different sources), a DEM, a GGM and GPS/levelling data. The available data are mostly unofficial and sparse (e.g., gravity, GPS and levelling data) or mostly based on global data sources (e.g., DEM and GGM).

During this research, a new gravity anomaly database (with contributions partly from ship-borne and satellite altimetry data) has been created by gathering all available data in the Iranian region from different local and international organizations, including data from the NCC and BGI. Different procedures were used for the refinement of the data (outlier detection) and also for taking into account the effect of topography in the gridding scheme.

Due to the lack of any high resolution photogrammetric based DEM in the region, a new DEM model (IRD04) was constructed with the resolution 15″ based on the 90 m high resolution SRTM DEM. A detail investigation was performed for studying the impact of using the high resolution SRTM data versus 1 km resolution models in the geoid modelling. The IRD04 model fitted the heights of the 260 GPS/levelling data at the level of 6 m.

Also, we make a detailed investigation of all available GGMs for choosing the best fitting model in the study area. From the results of this investigation we estimated that the GRACE model GGM02C extended with EGM96 fits the GPS/levelling data in Iran with 0.69 m and 7 ppm in the absolute and relative senses, respectively. This is the best model among all GGMs. It is also better compared to some local gravimetric geoid models. However, because of the interaction (correlation) between data in the least-squares scheme, in practice we got the same results by using the GRACE-GGM02S (satellite-only) model, which is theoretically preferred in the LSMS model.

The IRG04 gravimetric geoid model was evaluated in the absolute and relative senses, both internally by the estimation of the global MSE and externally by GPS/levelling data. The standard deviation of fitting between the IRG04 and the 35 precise GPS/levelling data is estimated to 0.29 m. However, because of the presence of different systematic errors in the observation and adjustment of the GPS/levelling and gravity data, the estimation of the accuracy in the absolute view usually can not show the real potential of a geoid model. However, it is well-known that GPS and levelling observations have very good accuracies in the relative sense, because most of the systematic errors are cancelled or eliminated through the differencing of observations. This lead us to evaluating the relative accuracy of the geoid models based on ΔH (GPS/geoid) versus levelling data (see Sect. 7.3.3). The results of this investigation shows that the IRG04 model fits the GPS/levelling with 3.8 ppm, which means that it is the best available geoid model in the study region.

Also, the influence on the geoid height stemming from a laterally density variation model was studied. We used a geological map to construct a rock density map of Iran. The numerical results show that the effect of using a lateral density model reaches up to 0.22 m, which is not negligible in a precise geoid determination with expected centimetre accuracy. Our results suggest that the effect of lateral topographical density variations is significant and ought to be taken into account specially in mountainous regions.

According to the various systematic errors that have been encountered in the terrestrial gravity and GPS/levelling data, we tried to find an optimal combination by combining the gravimetric and geometric geoid models based on the corrective surface idea. The RMS difference between of fitting the independent GPS/levelling data and the IRG04C combined model is 9 cm, which is almost 4 times better than the IRG04 gravimetric model. However, it should be mentioned here that the combined surface model is not an equipotential surface but it can be used in eliminating systematic errors and datum inconsistencies between the GPS and levelling height systems.

Based on this progress, the IRG04 geoid model combined with GPS observations can replace spirit levelling in most of engineering projects. This holds definitely in the relative form and preferably for short baselines.

Our main recommendations for further work are as follows:

1- According to the results of this research we cannot recommend further use of the current IfAG official gravimetric geoid model. We strongly recommend using the IRG04 geoid model in future projects.

2- One of the most important factors, that limit the accuracy of the gravimetric geoid in Iran, is the poor quality and quantity of the gravity data. It is evident that with the available distribution of gravity data (1 data per 65 km^2) it is almost impossible to obtain cm accuracy for the geoid. There are large mountains areas in Alborz and Zagros that do not have any gravity data at all. Most of these areas will not be surveyed for terrestrial gravity data in the near in future because of funding and logistics reasons. In mountainous and large countries like Iran it is strongly recommended to use airborne gravimetry to increase the quality and quantity of data.

3- For any future test it is recommended to use much denser high quality GPS/levelling points. Well distributed GPS/levelling data (especially in mountainous areas) is important in evaluating and refining the gravimetric geoid model.

6- Variance component estimation can be applied to the common adjustment of the heterogeneous heights (N, h and H). This leads to an in-depth analysis of the effects of correlation among heights of the same type and the intrinsic connection between the proper modelling of systematic errors and datum inconsistencies with the estimated variance components. It can be used for the optimum combination of these three parameters through the combined adjustment model. Unfortunately due to lack of information of covariance matrixes for h and H components we could not apply this approach in this research. The numerical studies including the scaling the

GPS/derived ellipsoidal height covariance matrix and evaluation of the accuracy of orthometric heights obtained from the national adjustments of levelling data is recommended for future works.

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Part Two

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PAPER A:

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