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PRODUCTIVITY ACROSS INDUSTRIES AND COUNTRIES: TIME SERIES THEORY AND EVIDENCE

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Productivity Across Industries and Countries: Time Series Theory and Evidence¹

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October, 1993

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Abstract

In this paper, we test whether aggregate productivity movements, especially convergence, are also reflected at the industry level. Using a new result on the asymptotic normality of panel unit root estimators, we find evidence for convergence in total factor productivity for sectors such as services and construction in 14 OECD countries from 1970-1987. However, surprisingly, we find that convergence does not hold for the manufacturing sector. Convergence in total industry occurs as a result of the declining share of manufacturing and the growing share of sevices in these countries.

KEY WORDS: Panel unit roots, economic growth, total factor productivity, convergence

1 Introduction

A key issue in understanding long-run economic growth is whether technology flows primarily between sectors within a nation or across countries within an industry: are there sector-specific or country-specific sources of productivity improvements? We use cross-section and time series techniques to study the movements of productivity levels in 14 OECD countries. We make use of industry-level data on value-added, capital stocks, and employment to construct total factor productivity from 1970-1987. The sectoral data are a valuable new resource in the empirical analysis of long-run growth.

The results at the industry level point out the importance of industry composition in empirical work on convergence across countries. Using a new result on asymptotic normality in estimating unit roots in panel data, we find that within sectors across countries, there is evidence for convergence for some industries, but not for others. These differences across sectors account for convergence at the national level. Although aggregate total factor productivity (TFP)¹ appears to be converging across OECD countries, this convergence is driven primarily by the non-manufacturing sectors of the economies. Within manufacturing we find only weak evidence for convergence over the period and find substantial evidence for divergence of productivity levels during the 1980's. Our results indicate that the lack of convergence found in larger samples of countries may be a consequence of varying sectoral composition in countries at different income levels.²

Largely due to a lack of data on labor and capital, almost all previous work on convergence across countries and regions has used real GDP per capita.³ Using cross-section regressions, Baumol (1986), Barro and Sala-i-

¹Through the paper TFP refers to log TFP.

²See Barro (1991) for evidence against unconditional convergence for large samples of countries.

³While convergence of labor productivity is predicted by many growth models, the use of output per capita instead of output per worker or per hour potentially confounds

Martin (1991, 1992) and Mankiw, Romer and Weil (1992) argue that countries and regions are converging, or catching-up, since initially poor areas grow faster than their richer counterparts. However, the cross-section evidence is not uniform. Barro (1991) and DeLong (1988) show that the particular sample of countries determines whether catch-up holds. Time series results on longer series for OECD countries also show evidence of common trends but no tendency for convergence in levels (for example, see Bernard and Durlauf 1991).

The neoclassical growth model without technology predicts convergence in output per worker for similar, closed economies based on the accumulation of capital. However, even in the neoclassical model, if the exogenous technology processes follow different long-run paths across countries, then there will be no tendency for output levels to converge. In this paper, we examine technological convergence by focussing on TFP. The results indicate that sectoral differences are important for understanding movements in aggregate income and productivity. Further work on longer samples and with more disaggregated data is needed to understand the relation between industry productivity, sectoral shifts, and aggregate variables.

1.1 Convergence in Aggregate TFP

The fundamental piece of evidence on cross-national growth in the OECD economies is that productivity and output differences have narrowed over time. The TFP movements are shown for 14 OECD countries from 1970-1987 in Figure 1.⁴ TFP has grown on average at a rate of 1.2% per year but the gap between the most productive country, the U.S. throughout the sample, and the least productive country declined consistently from 120% in

changes in participation rates with changes in productivity.

⁴The countries in the sample are U.S., Canada, Japan, Germany, France, Italy, U.K., Australia, the Netherlands, Belgium, Denmark, Norway, Sweden, and Finland. The U.S. is denoted by a "o" and Japan is denoted by a "+" in all Figures.

1970 to 85% in 1987. The overall decline in dispersion across countries can be seen in Figure 2 which plots the cross-sectional standard deviation of TFP. Dispersion has decreased from 17.5% to less than 13.5% during the period. Without the U.S. the dispersion is constant at about 13.5% until 1979 and drops steadily through the 1980's to 11% by 1987. The convergence in levels is also captured by regressing the average TFP growth rate for each country on the 1970 levels for TFP, as given in the following equation.⁵

$$\Delta TFP_{i} = \alpha + \beta TFP_{i}^{1970} + \varepsilon_{i}$$
(1)

$$coef \quad 0.125 \quad -0.0198 \quad \bar{R}^{2}: \ 0.4726$$

$$s.e. \quad (0.0224) \quad (0.0039)$$

The coefficient on the initial level is negative and significant, confirming the visual evidence from the cross-section.

1.2 Definitions of Convergence

Two distinct definitions of convergence have emerged in the empirical work. Cross-section analyses focus on the reduction in cross-sectional variance of output per worker. This idea of convergence as catching-up is linked to the predicted output paths from a neoclassical growth model with different initial levels of capital. Once countries attain their steady state levels of capital there is no further expected reduction in cross-section output variance. Time series studies define convergence as identical long-run trends, either deterministic or stochastic. This definition assumes that initial conditions do not matter within sample and tests for convergence using the framework of cointegration.

⁵Average growth rates here and throughout the paper are constructed as the trend coefficient from the regression of the log of TFP on a constant and a linear trend. This minimizes problems with measurement error and business cycle fluctuations.

Both these definitions have implications within our sample of advanced OECD countries from 1970-1987. If these 14 countries are on their long-run steady state growth paths as of 1970 then the appropriate framework for testing industry level convergence is that of common trends and cointegration. However, we consider TFP rather than output levels, so it is possible that capital had reached steady state values as of 1970 but technology was still being transferred from more productive to less productive countries within the sample. In Section 3, we will look for convergence largely within the time series framework. First, we discuss the sectoral data and evidence on cross-section standard deviations.

The rest of the paper is divided as follows: Section 2 describes the data and contains aggregate statistics on one digit industries; Section 3 presents a new result on testing for unit roots in panel data and empirical results on industries in the 14 OECD countries. Section 4 concludes and discusses future areas of research; Section 5 contains an appendix with the proof for Proposition 1.

2 Cross-Section Evidence

2.1 Data

The empirical work for this paper employs data on total factor productivity for (a maximum of) fourteen OECD countries and six sectors over the period 1970 to 1987. The fourteen countries are Australia, Belgium, Canada, Denmark, Finland, France, Italy, Japan, Netherlands, Norway, Sweden, U.K., U.S., and West Germany. The six sectors are Agriculture, Mining, Manufacturing, Electricity/Gas/Water (EGW), Construction, and Services. The basic data source is an updated version of the OECD Intersectoral Database (ISDB), constructed by Meyer-zu-Schlochtern (1988).⁶

⁶With the exception of the services aggregate, all the other sectors are taken directly from the ISDB. The services aggregate is constructed by summing Retail Trade,

For each country i, sector j, and year t, we construct a measure of the log of total factor productivity (TFP), designated $A_{ij}(t)$. This measure is constructed in the standard way, as a weighted average of capital and labor productivity, where the weights are the factor shares calculated assuming perfect competition and constant returns to scale. Details can be found in Meyer-zu-Schlochtern (1988). Our construction differs from his only in that our labor share is sector-specific, i.e. it is calculated as an average over time and country for each sector.

To summarize the data, Table 1 reports average annual TFP growth rates by country and sector for the period 1970 to 1987.⁷ Similarly, Figure 3 plots the log of TFP by sector for each country.

2.2 Industry TFP

Looking at the TFP levels by sector in Figure 3, we can see several immediate differences from the aggregate movements shown earlier. Sectors do not show the same patterns in either trend or dispersion over time and countries do not perform similarly across sectors. Manufacturing TFP grows on average at 1.9% per year and there is little change in the overall cross-section dispersion. Within manufacturing there are substantial differences as Japan grows at 4.0% per year and Norway at only 0.3%. On the other hand, EGW has no trend and there is substantial narrowing of the large initial gap between Japan and Norway during the sample.

The different sectoral contributions to aggregate TFP movements can be seen more clearly in Figure 4 which plots the cross-country sectoral standard deviations of TFP against time. Services and EGW show substantial evidence of catch-up, while at the other extreme, manufacturing has an overall increase in cross-country dispersion. Evidence on the other sectors is less

Transportation/Communication, F.I.R.E., and Other Services. Government Services are excluded.

⁷For a few sectors, 1986 is taken as the endpoint because of data availability.

clear-cut, construction falls initially and then steadies, mining rises dramatically and then falls back somewhat, while agriculture changes within years but shows little net change. These results do not change if the U.S. is removed from the sample. In fact the increase in manufacturing TFP dispersion is augmented.

The visual evidence on sectoral differences in TFP growth is dramatic. Sectors differ within and across countries. In particular, manufacturing appears to be leading to divergence in TFP and the aggregate levels are converging only because of the dramatic narrowing in other sectors, such as services.

3 Time Series Evidence

This section considers whether or not the provocative visual results found in the previous section can be supported with time series evidence. First, we will extend a recent advance in unit root econometrics by Levin and Lin (1992), and then we will apply this technique to the sectoral data in an attempt to test for convergence.

3.1 Testing for Unit Roots in Panel Data: Theory

The sectoral data we employ is available for a relatively short time horizon of eighteen years for most countries, 1970-1987. With such a limited sample of years, unit root testing would appear to be out of the question. However, a recent paper by Levin and Lin (1992) illustrates the relatively straightforward technique of testing for unit roots in panel data. Their basic findings are twofold: (1) that as both N and T go to infinity, the limiting distribution of the unit root estimator is centered and normal,⁸ and (2) that the panel

⁸Quah (1990) first noted this asymptotic normality result using a random fields data structure and rejected convergence of per capita output for a large cross-section of countries over 1960-1985. His estimator does not permit country-specific intercepts.

setting permits relatively large power improvements.

We consider the following general model with country-specific intercepts:

$$y_{it} = \mu_i + \rho y_{it-1} + \varepsilon_{it} \tag{2}$$

where the $\varepsilon_{it} \sim iid(0, \sigma_{\varepsilon}^2)$ and $\mu_i \sim iid(\bar{\mu}, \sigma_{\mu}^2)$. We also assume ε_{it} has $2 + \delta$ moments for some $\delta > 0$ and that $E\mu_i\varepsilon_{it} = 0$ for all i and t. Other standard regularity conditions are assumed to hold.

Let $\hat{\rho}$ and t_{ρ} be the OLS parameter estimate and *t*-statistic. Levin and Lin prove the following lemma:

Lemma 1 (Levin-Lin) Under the null hypothesis of a unit root with no drift, if N and T go to infinity with \sqrt{N}/T going to zero,

$$T\sqrt{N}(\hat{\rho} - (1 - \frac{3}{T})) \Rightarrow N(0, 10.2)$$
$$\sqrt{1.25}t_{\rho} + \sqrt{1.875N} \Rightarrow N(0, 1).$$

Furthermore, this result holds when a common time trend is included in the regression.

Several comments concerning this lemma are relevant. First, notice that when country-effects are included in the specification, a small-sample bias enters the distribution but disappears as T goes to infinity. This bias is independent of N and is analogous to the bias in standard panel data analysis described by Nickell (1981). Second, this leads the t-statistics to require a correction in order to be centered at zero: the uncorrected t-statistics are biased in the negative direction.

The Levin and Lin (1992) result provides asymptotic normality for the panel unit root tests in some common settings. One setting not considered in that paper is the case in which the data generating process is a unit root with nonzero drifts but time trends are omitted from the regression specification. West (1988) shows that asymptotic normality obtains for the N=1 case in this setting.⁹ The following proposition extends West's finding to the panel setting.

Proposition 1 Consider the regression model in Equation 2. Under the null hypothesis of a unit root with nonzero drifts $(\mu_i \neq 0)$,

$$\sqrt{N}T^{3/2}(\hat{\rho}-1) \Rightarrow N\left(0, \frac{7}{144} \frac{\sigma_{\epsilon}^2}{(\sigma_{\mu}^2 + \bar{\mu}^2)}\right).$$
(3)

Proof: See the Appendix.

This case differs substantially from that in Levin and Lin (1992). The asymptotic normality of $\hat{\rho}$ occurs as T goes to infinity because the results are driven by the time trends in y_{it} ; in constrast, the normality in the Levin-Lin proof is driven by the averaging across N non-normal distributions. Proposition 1 also has the advantage that it can be extended to allow for some dependence in the cross-section.

3.2 Evidence

To examine the convergence hypothesis while taking advantage of the time series aspect of the data, we focus on cross country deviations in TFP levels. Letting country 1 denote the benchmark country, our tests will be based on

$$DA_{ij}(t) \equiv A_{1j}(t) - A_{ij}(t), i = 2, ..., N.$$

Following Bernard and Durlauf (1991), we will say that country i is converging to country 1 if $DA_{ij}(t)$ is stationary. We do not necessarily require $A_{ij}(t)$

⁹Park and Phillips (1988) generalize this result substantially and show that asymptotic normality depends critically on the presence of only a single nonstationary regressor.

to exhibit a unit root with drift, although pretesting indicated that this null hypothesis could generally not be rejected.¹⁰

The cost of the short time horizon is that we cannot examine the hypothesis that only a subset of the fourteen countries are converging. That is, the panel test focuses on the extremes: we test the null hypothesis that all fourteen countries are converging against the alternative that as a group they are not converging. With the difficulty of constructing longer time series for TFP, we are unlikely to be able to test convergence in smaller groups of countries.

A related issue is how to choose the benchmark country. Asymptotically, of course, this choice should not matter, but in small samples it will be important. We report results when country 1 is chosen in two different ways: as the most productive country at the beginning of the sample, 1970, and as the median country in terms of productivity in 1970. Choosing country 1 as the most productive has the added advantage that we can construct a rough test of convergence in terms of the in-sample common trend: if productivity deviations from the most productive country exhibit a positive trend in sample, this would constitute strong evidence against the convergence hypothesis.

The results of our time series tests for convergence are reported in Table 2. Because of the small-sample bias problem, the reported t-statistics in the panel unit root regressions are not adjusted according to Levin and Lin (1992). Rather, critical values calculated using a Monte Carlo simulation with 500 repetitions are reported.

The first result of note pertains to the simple average trend in the productivity deviations from the most productive country. For total industry and for all sectors except mining, the average trends are negative. The presence

¹⁰When no time trends are included in the test for a unit root in the panel of TFP levels, the test fails to reject for every sectoral group. With a single common trend, the tests only reject the unit root null for the agricultural sector, but this sector exhibits a significant positive trend.

of these trends constitutes evidence in favor of the convergence hypothesis for all sectors except mining. Interestingly, however, the trend in the manufacturing sector is the smallest for the six sectors and the least significant of the negative trends. Furthermore, when Japan is excluded from the sample the trend is only slightly negative and is insignificantly different from zero.

Column (2) reports the results of the panel unit root tests when no time trends are included in the specification, as in Proposition 1. The reported estimates of ρ have been adjusted using the exact calculation of the bias according to Nickell (1981) and therefore should be centered at their true values.¹¹ It should be noted that the point estimates may be biased upward if there are deterministic trends in the deviations.

The point estimates for agriculture, mining, and EGW are all significantly less than unity, providing evidence against the unit root null in these sectors. The *t*-statistic for the construction sector and total industry also reject the null hypothesis of a unit root. In contrast, the results for manufacturing and services fail to reject the null. These statistics provide evidence for convergence in agriculture, EGW, construction, and total industry (less so for mining where the trend in column (1) is positive). For services and especially for manufacturing the results cannot reject the no convergence hypothesis.

Columns (3) and (4) reports results when the productivity deviations are taken from the country with the median level of productivity in 1970. Column (3) again uses the specification given in Proposition 1. Column (4) adds a common time trend as in Levin-Lin (1992). These results again highlight a

¹¹Nickell (1981) calculates the asymptotic bias as $N \rightarrow \infty$ for a fixed T to be

$$plim_{N\to\infty}\hat{\rho}-\rho = -\frac{(1+\rho)}{T-1}\left(1-\frac{1}{T}\frac{(1-\rho^T)}{1-\rho}\right)\left(1-\frac{2\rho}{(1-\rho)(T-1)}\left[1-\frac{1}{T}\frac{(1-\rho^T)}{1-\rho}\right]\right)^{-1}.$$

Given an estimated value of $\hat{\rho}$, this formula is used to solve for ρ , and the result is reported in column (2). Monte Carlo simulations suggest that this formula yields a good approximation even with N=14.

difference between the manufacturing sector and many of the other sectors, as manufacturing is the only sector with point estimates consistently greater than unity. Total industry shows evidence of convergence in the specification without a time trend, as does construction. Agriculture, construction and services reject the unit root null when a common time trend is added. Once again, however, a lack of power in the tests makes these results somewhat difficult to interpret.¹²

Overall, the panel/time series results are generally supportive of the graphical results documented using the cross-sectional data. Evidence in favor of convergence appears to be strongest in the non-manufacturing sectors and weakest in the manufacturing sector.

3.3 Movements in Sector Shares

To reconcile aggregate convergence with apparent lack of convergence in manufacturing, we examine movements in sectoral shares in total private output. Figure 5 plots the share of total private output for each of the six sectors. For Services, Agriculture, Construction, and Manufacturing, the trends are similar across countries. The share of manufacturing is declining in every country, as are the shares of agriculture and construction. Services is the only sector to show substantial share growth for all countries, accounting for at least 49% and as much as 64% of total industry output in 1987.

While services is growing as a share of output and manufactures is declining in all countries, there remain substantial differences in sectoral shares across countries. In particular, there is little tendency for shares to become more similar. Also, since all sectors except manufactures show convergence in productivity levels and the share of manufactures is declining, the convergence of total industry productivity is not surprising.

¹²Notice that the estimates of the general trend in the panel for each sector no longer has the convergence interpretation since these results are for deviations from the country with median instead of maximum productivity.

4 Conclusions and Future Research

In this paper, we show that aggregate productivity movements may disguise widely differing sectoral behavior. Across 14 OECD countries from 1970-1987, total industry productivity exhibits convergence with decreasing crosscountry variance. However, evidence from 6 sectors shows that aggregate convergence masks considerable differences. Manufacturing exhibits little convergence, the cross-section variance is actually increasing, while other sectors, such as services, show more catch-up. We extend existing results on asymptotic normality in panel unit root estimators in panel data and find additional evidence for differences across sectors. Manufacturing again shows least evidence for convergence and EGW the most.

Our examination of sectoral productivity movements in these OECD countries raises several interesting questions. To extend our analysis to discriminate between capital accumulation and technology accumulation as potential sources of convergence, we must do further work using more disaggregated data on output per worker. As suggested in the introduction and shown in last section, the role of changing sectoral composition influences aggregate TFP movements, especially in samples with countries at very different levels of development. The implications of sectoral composition for productivity movements in broader cross-sections of countries remains to be explored. Finally the methods employed in this paper may be applied to the analysis of productivity movements across the U.S states.

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5 Appendix: Proof of Proposition 1.

The OLS estimate of $\hat{\rho}$ can be written as

$$\hat{\rho} = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - \bar{y}_i)(y_{it-1} - \bar{y}_{i-1})}{\sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it-1} - \bar{y}_{i-1})^2}$$
(4)

where \bar{y}_i and \bar{y}_{i-1} denote the mean of \bar{y}_{it} and \bar{y}_{it-1} respectively. Substituting for y_{it} under the null hypothesis and normalizing appropriately, reveals

$$\sqrt{N}T^{3/2}(\hat{\rho}-1) = \frac{\frac{1}{\sqrt{N}}T^{-3/2}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1}-\bar{y}_{i-1})(\varepsilon_{it}-\frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it})}{\frac{1}{N}T^{-3}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1}-\bar{y}_{i-1})^2}$$
(5)

Now, consider the numerator and denominator separately. First, some algebra reveals that

$$y_{it-1} - \bar{y}_{i-1} = \mu_i \left(t - \frac{T+1}{2}\right) + \left(\tilde{y}_{it-1} - T^{-1} \sum_{t=1}^T \tilde{y}_{it-1}\right)$$
(6)

where a tilde is used to denote a variable that is I(1) with zero drift. Define

$$\xi_i = T^{-3/2} \sum_{t=1}^T (y_{it-1} - \bar{y}_{i-1}) (\varepsilon_{it} - \frac{1}{T} \sum_{t=1}^T \varepsilon_{it}).$$
(7)

Arguments such as those below reveal that the terms involving the expression $\frac{1}{T}\sum_{t=1}^{T} \varepsilon_{it}$ are $o_p(1)$, so that

$$\xi_{i} = T^{-3/2} \sum_{t=1}^{T} \mu_{i} t \varepsilon_{it} - \frac{1}{2} \mu_{i} T^{-1/2} \sum_{t=1}^{T} \varepsilon_{it}$$

$$- \frac{1}{2} \mu_{i} T^{-3/2} \sum_{t=1}^{T} \varepsilon_{it}$$

$$+ T^{-1/2} \left(T^{-1} \sum_{t=1}^{T} \tilde{y}_{it-1} \varepsilon_{it} - T^{-1} (T^{-1} \sum_{t=1}^{T} \tilde{y}_{it-1}) (\sum_{t=1}^{T} \varepsilon_{it} \right) + o_{p}(1).$$
(8)

The second to last term of this equation is $T^{-1/2}$ multiplied by a term that is asymptotically a demeaned Brownian motion, so that term is $o_p(1)$. Also, the third term of the equation involving $T^{-3/2} \sum_{t=1}^{T} \varepsilon_{it}$ is obviously $o_p(1)$. Then the results of West (1988) begin to apply:

$$T^{-3/2}\xi_{i} = T^{-3/2}\sum_{t=1}^{T}\mu_{i}t\varepsilon_{it} - \frac{1}{2}\mu_{i}T^{-1/2}\sum_{t=1}^{T}\varepsilon_{it} + o_{p}(1)$$
(9)
$$= \mu_{i}\left(T^{-1}\sum_{t=1}^{T}(\frac{t}{T})(T^{1/2}\varepsilon_{it}) - \frac{1}{2T}\sum_{t=1}^{T}T^{1/2}\varepsilon_{it}\right) + o_{p}(1)$$

$$\Rightarrow \mu_{i}\left(\int_{0}^{1}sdW(s) - \frac{1}{2}\int_{0}^{1}dW(s)\right)$$

$$= \mu_{i}\int_{0}^{1}(s - \frac{1}{2})dW(s)$$

$$= N(0, \frac{7}{12}\mu_{i}^{2}\sigma_{\varepsilon}^{2})$$

where σ_{ε}^2 is the variance of ε_{it} . Assuming that the μ_i are distributed i.i.d. with mean $\bar{\mu}$ and variance σ_{μ}^2 , the numerator behaves asymptotically like

$$\xi = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \xi_{i}$$

$$= N(0, \frac{7}{12} (\sigma_{\mu}^{2} + \bar{\mu}^{2}) \sigma_{\epsilon}^{2}).$$
(10)

Now consider the denominator of equation 5:

$$\frac{1}{N}T^{-3}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1}-\bar{y}_{i-1})^2 = \frac{1}{N}T^{-3}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it-1}^2 - T\sum_{i=1}^{N}\bar{y}_{i-1}^2\right)$$
(11)

which is naturally thought of as two terms. Once again, the results in West (1988) apply, so that the first term will behave asymptotically like a time trend:

$$T^{-3} \sum_{t=1}^{T} y_{it-1}^{2} = \sum_{t=1}^{T} (\mu_{i}(t-1) + \tilde{y}_{it-1})^{2}$$

$$= T^{-3} \sum_{t=1}^{T} \mu_{i}^{2}(t-1)^{2} + 2\mu_{i}T^{-3} \sum_{t=1}^{T} (t-1)\tilde{y}_{it-1} + T^{-3} \sum_{t=1}^{T} \tilde{y}_{it-1}^{2}$$
(12)

$$= \mu_i^2 T^{-3} \sum_{t=1}^T t^2 + o_p(1)$$

$$\Rightarrow \frac{1}{3} \mu_i^2.$$

Similarly, the second term is (ignoring the summation over N for the moment)

$$T^{-3}T\bar{y}_{i}^{2} = T^{-2}\bar{y}_{i-1}^{2} = T^{-2}\left(\frac{1}{T}\sum_{t=1}^{T}y_{it-1}\right)^{2}$$
(13)
$$= (T^{-2}\sum_{t=1}^{T}y_{it-1})^{2}$$

$$= (T^{-2}\sum_{t=1}^{T}\mu_{i}(t-1) + \tilde{y}_{it-1})^{2}$$

$$\Rightarrow \frac{1}{4}\mu_{i}^{2}.$$

By putting these two terms together, and summing across i the denominator becomes

$$Denom \Rightarrow \frac{1}{12}E\mu_i^2 = \frac{1}{12}(\sigma_{\mu}^2 + \bar{\mu}^2).$$
 (14)

Finally, combine the results for the numerator and denominator, and the proposition is proven:

$$\sqrt{N}T^{3/2}(\hat{\rho} - 1) = \frac{N(0, \frac{7}{12}(\sigma_{\mu}^{2} + \bar{\mu}^{2})\sigma_{\varepsilon}^{2})}{\frac{1}{12}(\sigma_{\mu}^{2} + \bar{\mu}^{2})}$$

$$= N(0, \frac{7}{144}(\sigma_{\mu}^{2} + \bar{\mu}^{2})^{-1}\sigma_{\varepsilon}^{2}).$$
Q.E.D.

Table 1													
Average Growth Rates of Total Factor Productivity													
Country	Agric.	Mining	Mfg.	E/G/W	Constr.	Svs.	Tot. Ind.						
U.S.	0.015	-0.037	0.016	0.006	-0.020	0.002	0.003						
Canada	0.009	-0.060	0.008	0.006	0.014	0.006	0.004						
Japan	-0.002	0.018	0.040	-0.012	-0.022	na	0.015						
W. Germany	0.043	-0.028	0.015	0.003	0.007	na	0.013						
France	0.040	-0.036	0.017	0.027	0.009	0.012	0.017						
Italy	0.020	na	0.029	-0.017	-0.017	na	0.010						
U.K.	0.036	-0.015	0.012	0.007	-0.006	0.005	0.009						
Australia	0.018	-0.008	0.014	0.019	0.008	na	0.005						
Netherlands	0.044	0.013	0.024	-0.012	-0.010	na	0.013						
Belgium	0.037	-0.012	0.035	0.028	0.011	0.005	0.016						
Denmark	0.041	0.082	0.019	0.032	-0.011	0.010	0.014						
Norway	0.021	0.074	0.003	0.004	-0.003	0.007	0.015						
Sweden	0.020	-0.037	0.011	0.022	0.021	0.009	0.012						
Finland	0.022	0.019	0.026	0.014	0.015	0.013	0.017						
AVERAGE	0.026	-0.002	0.019	0.009	-0.000	0.008	0.012						

Table 2														
Time Series Tests for Convergence														
	D	eviations	s from M	ost	Deviations from Median									
	Prod	uctive C	ountry in	n 1970	Productive Country in 1970									
	(1)		(2)		(3)		(4)							
Sector	Trend	TStat	ρ	TStat	ρ	TStat	ρ	TStat						
Agriculture	0119	-7.54	.742	-7.60**	.846	-5.60	.808	-6.95**						
Mining	.0159	2.84	.876	-6.98**	.970	-4.06	.953	-4.78						
Manufacturing	0033	-3.58	1.083	-2.99	1.141	-3.37	1.091	-2.36						
Elec/G/W	0226	-14.54	.951	-6.24**	.914	-4.73	.936	-5.10						
Construction	0207	-18.59	1.0235	-6.84**	.869	-7.65**	.977	-5.80**						
Services	0068	-17.02	1.186	-1.25	.846	-4.06	.770	-5.90**						
Total Industry	0097	-25.35	1.113	-4.04*	.971	-5.30*	1.0844	-2.54						

<u>Notes</u>: Column (1) reports estimates of the average trend in the panel. Columns (2)-(3) report panel unit root tests based on Proposition 1 in the paper, i.e. excluding time trends from the specification. Column (4) includes a single common trend and is based on Levin and Lin (1992). All regressions include country-specific intercepts.

The ρ estimate in columns (2) and (3) is adjusted using the exact bias formula in Nickell (1981), and the ρ estimates in (4) are adjusted by 3/T: the bias according to Levin and Lin under the null of a unit root. If the true ρ is less than one, the point estimates in (4) will be biased upward; the t-statistics, however, remain correct.

Critical values for the t-statistics were tabulated using a Monte Carlo simulation with 500 iterations, and significance levels are indicated in the table by asterisks: 10% (*) and 5% (**). For the Monte Carlo simulation for each sector, TFP levels by country were differenced and then means and standard deviations of these first differences were used to generate the data for the Monte Carlo with N=14 and T=18.



Total Industry



Figure 2



Figure 3













Figure 4













Figure 5





















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