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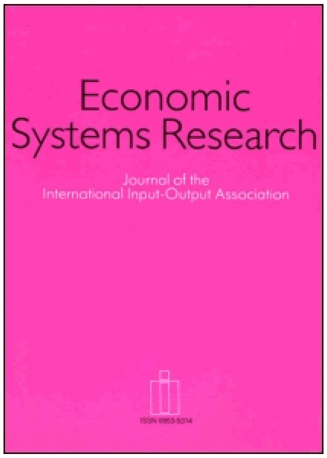

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Structural Decomposition Techniques: Sense and Sensitivity

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Structural Decomposition Techniques: Sense and Sensitivity

ERIK DIETZENBACHER & BART LOS

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ABSTRACT *Structural decomposition techniques are widely used to break down the growth in some variable into the changes in its determinants. In this paper, we discuss the problems caused by the existence of a multitude of equivalent decomposition forms which are used to measure the contribution of a specific determinant. Although it is well known that structural decompositions are not unique, the extent of the problem and its consequences seem to have been largely neglected. In an empirical analysis for The Netherlands between 1986 and 1992, results are calculated for 24 equivalent decomposition forms. The outcomes exhibit a large degree of variability across the different forms. We also examine the two approaches that have been used predominantly in the literature. The average of the two so-called polar decompositions appears to be remarkably close to the average of the full set of 24 decompositions. The approximate decomposition with mid-point weights appears to be almost exact. Although this last alternative might seem a solution to the problem of the marked sensitivity, in fact, it only conceals the problem.*

KEYWORDS: *Decomposition techniques, input–output framework, sensitivity analysis*

1. Introduction

Structural decomposition techniques have become a major tool for disentangling the growth in some variable over time, separating the changes in the variable's constituent parts (see Rose & Casler, 1996, for a detailed review of the literature). Within an input–output (IO) framework, the analysis of changes in the structure of production has a long tradition, dating back to Leontief (1953) (see, for example, Chenery *et al.*, 1962; Vaccara & Simon, 1968; Carter, 1970; Leontief & Ford,

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1972; Stäglin & Wessels, 1972). In the 1980s, this type of analysis witnessed a remarkable revival, with seminal contributions by Wolff (1985), Feldman *et al.* (1987) and Skolka (1989). Structural decomposition analysis is defined 'as a method of distinguishing major shifts within an economy by means of comparative static changes in key sets of parameters' (Skolka, 1989, p. 46).[1]

Using structural decomposition techniques allows for the quantification of the underlying sources of change in a wide variety of variables. Examples are output (Fujimagari, 1989), value added (Oosterhaven *et al.*, 1995), energy use (Lin & Polenske, 1995), labour requirements (Forssell, 1990), volume of imports (Kanemitsu & Ohnishi, 1989), output of services industries (Barker, 1990) and total input requirements (Afrasiabi & Casler, 1991), all at the sectoral level.

The methodology of structural decomposition analysis is similar to that of growth accounting, where the objective is to break down the growth rate of aggregate output into contributions from the growth of input and the growth of technology (see, for example, Solow, 1957; Kendrick, 1961; Denison, 1974, 1985). Contributions that combine IO elements with a growth accounting set-up include Wolff (1985, 1994), Galatin (1988), Fontela (1989) and Wolff and Howell (1989), who examine the decomposition of total factor productivity growth.[2] Other areas where similar techniques are used, and to which our results carry over, include demographic accounting and shift and share analysis (see, for example, Oosterhaven, 1981).

To sketch the typical result in an empirical decomposition analysis, consider the following case. Let the change in sectoral output levels be decomposed into several sources, one of which is the change in the matrix of technical coefficients. A characteristic outcome would be that, for sector i, the contribution of technical changes to the output change might be 60%. A major problem of structural decomposition techniques, however, is that the decomposition is not unique. This problem has been recognized and analyzed in detail for a decomposition with only two determinants.[3] For this specific case, the problem has been 'solved' on an *ad hoc* basis, by taking the average. This 'solution', which has certain intuitively appealing properties, is somewhat misleading, however, in the sense that it only applies to the simplest case of two determinants. As a consequence, for the economically more meaningful decompositions with a larger number of determinants, the non-uniqueness problem, its extent and its implications seem to have been largely neglected.

In the next section, we show that, when the number of determinants or sources is n, the number of equivalent decomposition forms is $n!$ In Section 3, we present the results of an empirical analysis in which $n = 4$. It turns out that the outcomes are very sensitive to the specific decomposition. For the previously mentioned example of a 60% contribution of technical changes, we would find outcomes ranging from 50% to 70%, for example. Although this range may not seem to be extremely large, it seriously affects the economic interpretation of the results. If, on the one hand, the contribution of technical changes were 50%, then the contribution of all other sources would also be 50%. The contribution of technical changes is then judged as being equally important to the contributions of all other sources. If, on the other hand, the contribution of technical changes were 70%, then the contribution of the other sources would be only 30%. In other words, the contribution of technical changes is twice as great as the contribution of all other sources. All this implies that measuring the contribution of factors such as technical changes crucially depends on the way that this contribution is measured. A large sensitivity causes a serious problem, since there is no reason why one decomposition should be preferred to the others on theoretical grounds. Therefore, our sensitivity results cast doubts on the sense of structural decomposition techniques for the purpose of splitting the growth in some variable into its determinants.

Section 4 discusses the two *ad hoc* 'solutions' that have been applied widely. First, we empirically compare the average of the two so-called polar decompositions with the average of the full set of decompositions. The outcomes appear to be remarkably close to each other. Second, we examine the 'solution' of using mid-point weights, which is a special case of the approximate decompositions. The decompositions in Sections 2 and 3 are all exact, in the sense that the sum of the contributions equals 100%. Approximate decompositions are based on the discrete approximation of the unique decomposition in continuous time, and the contributions do not generally sum to 100%. The empirical results indicate that the errors may become very small, however. Clearly, this does not imply a solution to the non-uniqueness problem; it only adds another possibility to the $n!$ exact decomposition forms.

2. The Methodology

The problem that is addressed in this paper is caused by the existence of a multitude of equivalent decompositon forms. In order to sketch the problem, consider the simplest case first. In other words, let $y = xz$, where y, x and z are scalars, vectors and/or matrices. The change in y between two points in time, i.e. $\Delta y = y(1) - y(0)$, may be decomposed as follows:

$$\Delta y = (\Delta x)z(1) + x(0)(\Delta z) \tag{1}$$

$$= (\Delta x)z(0) + x(1)(\Delta z) \tag{2}$$

In this simple case, there are two alternative ways of additively decomposing the change in y into the changes in its determinants. The decompositions in equations (1) and (2) are equivalent and there is no reason why one decomposition should be preferred in favour of the other. For both equations (1) and (2), the components are typically described as '*the* contribution of the change in x (respectively z) to the change in y'. A common 'solution' to the existence of several equivalent decomposition forms is to take the mean of the expressions. This yields

$$\Delta y = (\Delta x)z(\tfrac{1}{2}) + x(\tfrac{1}{2})(\Delta z) \tag{3}$$

where, for example $z(\frac{1}{2}) = \frac{1}{2}z(0) + \frac{1}{2}z(1)$.[4] Note that this 'solution' is very attractive. It is exact (i.e. the contributions on the right-hand side sum to Δy) and it is intuitively appealing, in the sense that both Δ terms have the same type of weights and, moreover, have mid-point weights. Unfortunately, this 'solution' is only possible in the simplest case with two determinants.

In the general case, we have

$$y = x_1 x_2 \dots x_n \tag{4}$$

In deriving the additive decomposition of Δy, we may start at one end, which yields

$$\begin{aligned}\Delta y = {} & (\Delta x_1)x_2(1)x_3(1)\dots x_{n-1}(1)x_n(1) + x_1(0)(\Delta x_2)x_3(1)\dots x_{n-1}(1)x_n(1) + \\ & \vdots \\ & + x_1(0)x_2(0)x_3(0)\dots(\Delta x_{n-1})x_n(1) + x_1(0)x_2(0)x_3(0)\dots x_{n-1}(0)(\Delta x_n)\end{aligned} \tag{5}$$

Starting at the other end yields

$$\begin{aligned}\Delta y = {} & (\Delta x_1)x_2(0)x_3(0)\ldots x_{n-1}(0)x_n(0) + x_1(1)(\Delta x_2)x_3(0)\ldots x_{n-1}(0)x_n(0) + \\ & \vdots \\ & + x_1(1)x_2(1)x_3(1)\ldots(\Delta x_{n-1})x_n(0) + x_1(1)x_2(1)x_3(1)\ldots x_{n-1}(1)(\Delta x_n)\end{aligned} \tag{6}$$

Although equations (5) and (6) are the most convenient expressions from a notational point of view, there is no reason why we should start at one end or at the other. All equivalent decomposition forms are obtained by applying equation (5) to each permutation of the set $\{1, \ldots, n\}$ of indices, and rewriting the n additive components in their original ordering as in equation (4). Thus, the number of different decomposition forms equals the number of permutations, which is (n!).[5] Equations (5) and (6) are termed 'polar decompositions', because they work through the original ordering $\{1, \ldots, n\}$ from left to right and from right to left.

It is well known that structural decompositions are non-unique and 'as a result, measures for various sources of change are not unique' (Rose & Casler, 1996, p. 47). It is somewhat surprising, therefore, that no attention has been paid to investigating the seriousness of this consequence.[6] Instead, most authors adopt the *ad hoc* 'solution' of either taking the average of the two polar decompositions in equations (5) and (6), or using equation (5) with mid-point weights. Recall that, in the simplest case of two determinants, this approach yields the same result (see equation (3)). It should be emphasized that taking the average of the two polar decompositions in the general case is still exact but not as intuitively appealing as the 'solution' in equation (3). The Δ terms have a complex weighting structure; moreover, they do not have the same type of weights. The *ad hoc* 'solution' of applying mid-point weights yields a decomposition that is not exact.

In the next section, we analyze the variability of the outcomes obtained from the n! different decomposition forms. Using an IO framework, we consider the sectoral changes in the labour costs and in the imports. Based on a 214-sector IO table, the model is

$$\mathbf{w} = \hat{\mathbf{u}}\mathbf{q} \tag{7}$$

$$\mathbf{m} = \hat{\mathbf{v}}\mathbf{q} \tag{8}$$

$$\mathbf{q} = \mathbf{A}\mathbf{q} + \mathbf{B}\mathbf{f} \tag{9}$$

with

- **w** the 214×1 vector of sectoral labour costs (wages and salaries, including employers' contribution)
- **m** the 214×1 vector of sectoral imports
- **q** the 214×1 vector of sectoral output levels
- **u** the 214×1 vector of sectoral labour costs per unit of this sector's output (in money terms)[7]
- **v** the 214×1 vector of sectoral imports per unit of output
- **A** the 214×214 matrix of technical coefficients a_{ij}, measuring the input from sector i in sector j, per unit of sector j's output
- **B** the 214×5 matrix of bridge coefficients b_{ik}, measuring the fraction of the final demand in category k that is spent on products from sector i, describing the final demand mix or distribution
- **f** the 5×1 vector with total final demands in each of the five categories, i.e. private consumption, government consumption, exports, investments, and imputed bank services

The solution of the model, which is the basis for the decompositions, is given as

$$\mathbf{w} = \hat{\mathbf{u}}\mathbf{LBf} \tag{10}$$

$$\mathbf{m} = \hat{\mathbf{v}}\mathbf{LBf} \tag{11}$$

where $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ denotes the Leontief inverse, describing the total input requirements. According to equation (10), the change $\Delta\mathbf{w}$ in sectoral labour costs may be decomposed into four components:

(1) the effects of the change $\Delta\hat{\mathbf{u}}$ in the labour costs per unit;
(2) the effects of technical changes $\Delta\mathbf{L}$;[8]
(3) the effects of changes $\Delta\mathbf{B}$ in the final demand mix;
(4) the effects of the changes $\Delta\mathbf{f}$ in the final demand levels.[9]

3. Sensitivity Analysis

Based on equations (10) and (11), the decompositions for $\Delta\mathbf{w}$ and $\Delta\mathbf{m}$ were applied to the 214-sector IO tables in current prices of The Netherlands, for the years 1986 and 1992.[10] Equations (10) and (11) include four explanatory terms, so that the number of different decomposition forms is 24. The two polar forms were derived in equations (5) and (6) for the general case. For the present application, the two polar decompositions of $\Delta\mathbf{w}$, for example, read as follows:

$$\begin{aligned}\Delta\mathbf{w} &= \mathbf{w}(92) - \mathbf{w}(86) \\ &= \hat{\mathbf{u}}(92)\mathbf{L}(92)\mathbf{B}(92)\mathbf{f}(92) - \hat{\mathbf{u}}(86)\mathbf{L}(86)\mathbf{B}(86)\mathbf{f}(86)\end{aligned} \tag{12}$$

$$\begin{aligned}&= (\Delta\hat{\mathbf{u}})\mathbf{L}(92)\mathbf{B}(92)\mathbf{f}(92) + \hat{\mathbf{u}}(86)(\Delta\mathbf{L})\mathbf{B}(92)\mathbf{f}(92) \\ &\quad + \hat{\mathbf{u}}(86)\mathbf{L}(86)(\Delta\mathbf{B})\mathbf{f}(92) + \hat{\mathbf{u}}(86)\mathbf{L}(86)\mathbf{B}(86)(\Delta\mathbf{f})\end{aligned} \tag{13}$$

$$\begin{aligned}&= (\Delta\hat{\mathbf{u}})\mathbf{L}(86)\mathbf{B}(86)\mathbf{f}(86) + \hat{\mathbf{u}}(92)(\Delta\mathbf{L})\mathbf{B}(86)\mathbf{f}(86) \\ &\quad + \hat{\mathbf{u}}(92)\mathbf{L}(92)(\Delta\mathbf{B})\mathbf{f}(86) + \hat{\mathbf{u}}(92)\mathbf{L}(92)\mathbf{B}(92)(\Delta\mathbf{f})\end{aligned} \tag{14}$$

The four components on the right-hand side of equations (13) and (14) describe the contribution to $\Delta\mathbf{w}$ of the effects of changes in the labour costs per unit, technical changes, changes in the final demand mix and changes in the final demand levels. In the tables, these components are abbreviated and denoted as the $\Delta\hat{\mathbf{u}}$ effects, the $\Delta\mathbf{L}$ effects, the $\Delta\mathbf{B}$ effects and the $\Delta\mathbf{f}$ effects.

3.1. The Variability of Outcomes

Table 1 presents the contributions of the four effects for the 10 'most important' sectors. In other words, we have selected the five sectors with the largest growth percentages in labour costs and the five sectors with the largest absolute increases in labour costs. For example, for sector 156, the smallest $\Delta\hat{\mathbf{u}}$ effect found in the 24 different decompositions is 8.2 (million Dutch guilders); the largest $\Delta\hat{\mathbf{u}}$ effect is 21.4. The next column gives the range r_i as the difference between $\max_i$ and $\min_i$. The average $\Delta\hat{\mathbf{u}}$ effect over the 24 decompositions is reported in the column under μ_i as 14.4.[11] The standard deviation is given in the column under σ_i as 5.2. The last two columns relate the range and the standard deviation to the mean,

Table 1. Results for the 10 'most important' sectors

Sector[a]	$\min_i$	$\max_i$	r_i	μ_i	σ_i	r_i/μ_i^b	σ_i/μ_i^b
Sectors with the largest percentage growth							
$i=156$	$w_i(86)=539$, $w_i(92)=1436$, $\Delta w_i=897$, $\Delta\%=166.4^c$						
$\Delta\hat{\mathbf{u}}$	8.2	21.4	13.3	14.4	5.2	92.2	36.3
$\Delta\mathbf{L}$	392.5	517.3	124.8	454.2	58.1	27.5	12.8
$\Delta\mathbf{B}$	176.7	238.8	62.0	207.2	27.5	29.9	13.3
$\Delta\mathbf{f}$	142.9	301.1	158.2	221.2	67.6	71.5	30.6
$i=153$	$w_i(86)=1245$, $w_i(92)=2842$, $\Delta w_i=1597$, $\Delta\%=128.3$						
$\Delta\hat{\mathbf{u}}$	119.1	248.1	129.0	180.4	50.7	71.5	28.1
$\Delta\mathbf{L}$	584.4	822.9	238.5	700.0	98.0	34.1	14.0
$\Delta\mathbf{B}$	209.2	301.7	92.5	253.4	34.3	36.5	13.5
$\Delta\mathbf{f}$	333.2	599.3	266.0	463.3	108.6	57.4	23.5
$i=205$	$w_i(86)=77$, $w_i(92)=172$, $\Delta w_i=95$, $\Delta\%=123.4$						
$\Delta\hat{\mathbf{u}}$	1.3	2.8	1.5	2.0	0.6	76.6	32.7
$\Delta\mathbf{L}$	0	0	0	0	0	—	—
$\Delta\mathbf{B}$	50.8	68.4	17.6	59.6	8.7	29.5	14.5
$\Delta\mathbf{f}$	24.9	42.1	17.1	33.5	8.6	51.2	25.6
$i=127$	$w_i(86)=189$, $w_i(92)=416$, $\Delta w_i=227$, $\Delta\%=120.1$						
$\Delta\hat{\mathbf{u}}$	16.1	32.6	16.5	23.9	6.6	69.3	27.7
$\Delta\mathbf{L}$	19.0	27.4	8.4	23.1	3.5	36.2	15.2
$\Delta\mathbf{B}$	83.1	120.3	37.1	101.3	16.4	36.7	16.2
$\Delta\mathbf{f}$	59.3	99.3	40.0	78.8	17.0	50.7	21.5
$i=157$	$w_i(86)=1032$, $w_i(92)=2249$, $\Delta w_i=1217$, $\Delta\%=117.9$						
$\Delta\hat{\mathbf{u}}$	5.0	10.8	5.8	7.8	2.2	75.2	29.0
$\Delta\mathbf{L}$	451.6	578.2	126.6	514.6	62.3	24.6	12.1
$\Delta\mathbf{B}$	285.0	355.7	70.7	320.1	34.1	22.1	10.7
$\Delta\mathbf{f}$	279.2	470.0	190.8	374.5	80.8	50.9	21.6
Sectors with the largest absolute growth							
$i=121$	$w_i(86)=13\,212$, $w_i(92)=20\,712$, $\Delta w_i=7500$, $\Delta\%=56.8$						
$\Delta\hat{\mathbf{u}}$	1902.6	2607.2	704.6	2249.5	310.2	31.3	13.8
$\Delta\mathbf{L}$	387.9	614.1	226.2	492.8	70.9	45.9	14.4
$\Delta\mathbf{B}$	597.3	911.5	314.2	745.0	107.0	42.2	14.4
$\Delta\mathbf{f}$	3606.2	4428.4	822.2	4012.7	341.4	20.5	8.5
$i=123$	$w_i(86)=7726$, $w_i(92)=12\,225$, $\Delta w_i=4499$, $\Delta\%=58.2$						
$\Delta\hat{\mathbf{u}}$	1235.6	1685.6	449.9	1458.5	215.1	30.8	14.7
$\Delta\mathbf{L}$	6.6	9.2	2.6	7.8	0.7	33.7	9.6
$\Delta\mathbf{B}$	244.9	375.7	130.7	308.1	55.4	42.4	18.0
$\Delta\mathbf{f}$	2482.3	2971.0	488.7	2724.6	224.3	17.9	8.2
$i=146$	$w_i(86)=5385$, $w_i(92)=8232$, $\Delta w_i=2847$, $\Delta\%=52.9$						
$\Delta\hat{\mathbf{u}}$	363.0	519.9	156.9	439.8	66.8	35.7	15.2
$\Delta\mathbf{L}$	291.9	405.5	113.7	346.3	42.2	32.8	12.2
$\Delta\mathbf{B}$	300.5	425.0	124.5	360.3	47.3	34.6	13.1
$\Delta\mathbf{f}$	1564.6	1838.9	274.3	1700.6	104.9	16.1	6.2
$i=171$	$w_i(86)=8221$, $w_i(92)=10\,863$, $\Delta w_i=2642$, $\Delta\%=32.1$						
$\Delta\hat{\mathbf{u}}$	1169.0	1525.4	356.4	1341.4	130.0	26.6	9.7
$\Delta\mathbf{L}$	−7.9	−5.6	2.3	−6.7	0.7	−34.8	−10.6
$\Delta\mathbf{B}$	−714.4	−505.8	208.6	−607.2	87.5	−34.4	−14.4
$\Delta\mathbf{f}$	1723.4	2117.1	393.7	1914.4	131.1	20.6	6.8
$i=162$	$w_i(86)=6933$, $w_i(92)=9417$, $\Delta w_i=2484$, $\Delta\%=35.8$						
$\Delta\hat{\mathbf{u}}$	654.2	856.1	201.9	751.9	82.0	26.9	10.9
$\Delta\mathbf{L}$	−279.1	−177.7	101.4	−226.1	40.2	−44.9	−17.8
$\Delta\mathbf{B}$	205.3	306.7	101.5	252.3	30.1	40.2	11.9
$\Delta\mathbf{f}$	1578.1	1843.2	265.2	1706.0	86.1	15.5	5.0

[a]The sectors are as follows: 156, economic advising agencies; 153, computer services; 205, gambling and betting services; 127, beverage serving services (no accommodation); 157, other business services; 121, wholesale trade; 123, retail trade; 146, railways, communication services, taxi and coach enterprises; 171, special (primary) education (for handicapped children); 162, local government.

[b]As a percentage, i.e. $100(r_i/\mu_i)$ and $100(\sigma_i/\mu_i)$.

[c]The percentage growth, i.e. $\Delta\% = 100[\Delta w_i/w_i(86)]$.

Table 2. Percentage contributions for sector 127

Labour costs	$\Delta\hat{\mathbf{u}}$	$\Delta\mathbf{L}$	$\Delta\mathbf{B}$	$\Delta\mathbf{f}$
min	7	8	37	26
max	14	12	53	44
(*)	9	12	53	26
(**)[a]	11	9	37	44

[a]Because of rounding, the numbers do not sum to 100.

both as a percentage. For example, for sector 156, the range and the standard deviation amount to 92.2% and 36.3% of the average effect μ_i.

The results indicate that there is substantial variation in the outcomes of the 24 different decomposition forms. For the upper part of Table 1, with the sectors that have the largest percentage growth, the $\Delta\hat{\mathbf{u}}$ effect and the $\Delta\mathbf{f}$ effect show the greatest variability. On average, the ratio between the range and the mean is 50% for the effects in the upper part. For the results in Table 1, as well as for most of the other sectoral calculations that we have carried out, the following 'rule of thumb' seems to apply. Roughly speaking, a value of $100(r_i/\mu_i)$ that equals 50 implies that the minimum observation is $0.75\mu_i$ and the maximum observation is $1.25\mu_i$. The average r_i/μ_i ratio in the lower part of Table 1, with sectors that grew the most in an absolute sense, is lower, i.e. 31%. In part, this is caused by the sometimes exceptionally large values for μ_i. Similar results were also found for the decomposition of the growth $\Delta\mathbf{m}$ in the sectoral imports.

To indicate the implications for the economic interpretation of the results, we have taken sector 127 (beverage serving services) as an example. Table 2 reports the percentage contributions of each of the four effects to the change in this sector's labour costs. The rows min and max give the smallest and largest contributions of the $\Delta\hat{\mathbf{u}}$ effect, for example, that was found. The numbers are readily obtained from Table 1. At first sight, the differences do not seem to be dramatically large; the range for the $\Delta\mathbf{B}$ effect is 16%, while it is 18% for the $\Delta\mathbf{f}$ effect. It may be expected, however, that the one effect takes its maximum position when the other effect is at its minimum.

This is precisely what happens, as reflected by the rows (*) and (**). These rows present the results for two of the 24 decompositions. In the case of (*), one would have concluded that the $\Delta\mathbf{B}$ effect was twice as large as the $\Delta\mathbf{f}$ effect. If (**) were used, however, the the $\Delta\mathbf{B}$ effect would have been reported to be clearly smaller than the $\Delta\mathbf{f}$ effect.

Table 3 summarizes the variability of the outcomes of the 24 different decomposition forms for all sectors. Consider, for example, the decomposition of the change in the sectoral labour costs. For each sector i $(=1,\ldots,214)$, the $\Delta\hat{\mathbf{u}}$ effect was calculated for all 24 decomposition forms, yielding $100(r_i/\mu_i)$ and $100(\sigma_i/\mu_i)$. The figures in Table 3 report the average and the standard deviation, taken over all the sectors, of the absolute values.[12]

The results in Table 3 clearly show that the outcomes of the 24 different decomposition forms exhibit considerable variability.[13] The average variation coefficient (σ_i/μ_i) for the $\Delta\mathbf{L}$ and the $\Delta\mathbf{B}$ effect is between 20% and 25% for the decomposition of both the sectoral labour costs and the sectoral imports. Note also that there is considerable variability in the variation coefficients across the sectors, as reflected by the size of the standard deviations. This holds, in particular, for the variation coefficients of the $\Delta\mathbf{B}$ effect. A large standard deviation over the sectors

Table 3. Average variability

	$100(r_i/\mu_i)$		$100(\sigma_i/\mu_i)$	
	Average	Standard deviation	Average	Standard deviation
Labour costs				
$\Delta\hat{\mathbf{u}}$	42.7	16.2	15.7	6.0
$\Delta\mathbf{L}$	54.9	42.4	20.2	16.9
$\Delta\mathbf{B}$	60.4	102.3	23.4	43.9
$\Delta\mathbf{f}$	31.0	21.5	10.7	7.7
Imports				
$\Delta\hat{\mathbf{v}}$	42.7	16.2	15.7	6.0
$\Delta\mathbf{L}$	61.5	48.4	21.7	19.5
$\Delta\mathbf{B}$	67.1	97.1	24.8	42.1
$\Delta\mathbf{f}$	37.6	33.6	14.1	14.3

indicates that quite a few sectors have a variation coefficient that is almost zero, while there are also several sectors with a very large variation coefficient. Hence, for some sectors, the $\Delta\mathbf{B}$ effect hardly varies between the 24 different decomposition forms; in contrast, for other sectors, the variation is extremely large.

Since the application of the structural decomposition technique induces a multitude of different computational forms, and since all these forms are equivalent, in the sense that no form is to be preferred on theoretical grounds to the others, we advocate that the average is computed. However, since the results may differ greatly between the different forms, we feel that the range (or the standard deviation) also provides relevant information. Therefore, we suggest that, in empirical analyses, the average effects and the ranges (or the standard deviations) are published.

3.2. The Effects of Aggregation

The next experiment addresses the question of whether the variability of the outcomes for the different decomposition forms is affected by the level of aggregation. To this end, the calculations that were carried out at the 214-sector level, and which were summarized in Table 3, were repeated for a 113-sector, a 59-sector, a 27-sector and the one-sector classification.[14] Our 59-sector classification closely resembles the classification used by Statistics Netherlands in its official publications (see, for example, Statistics Netherlands, 1995). The one-sector case covers the utmost aggregation, where the entire production process constitutes the only sector. The 113-sector and the 27-sector classifications were constructed somewhat arbitrarily for the present purpose.

In obtaining the results, the absolute values of the ratio r_i/μ_i (as a percentage) were first calculated for each sector i. The averages over the sectors and the standard deviations are presented in Table 4. On the whole, the results exhibit a slowly declining variability, as reflected by decreasing average r_i/μ_i values. It should be noted, however, that it is possible that aggregation incidentally increases the variability. Observe that the variability of the outcomes remains substantial, even in the utmost one-sector case, for the $\Delta\hat{\mathbf{u}}$, the $\Delta\hat{\mathbf{v}}$ and both $\Delta\mathbf{L}$ effects. In conclusion, it is not true that the variability vanishes or even reduces drastically as a result of the aggregation of the original data.

Table 4. The effects of aggregation before decomposition[a]

	No. of sectors				
	214	113	59	27	1
Labour costs					
$\Delta\hat{\mathbf{u}}$	42.7	39.5	41.2	35.0	24.3
	(16.2)	(12.2)	(13.9)	(8.7)	(—)[b]
$\Delta\mathbf{L}$	54.9	56.1	57.5	49.7	36.5
	(42.4)	(39.7)	(40.3)	(20.4)	(—)
$\Delta\mathbf{B}$	60.4	52.9	44.8	43.8	—[c]
	(102.3)	(40.1)	(17.6)	(13.6)	(—)
$\Delta\mathbf{f}$	31.0	27.4	31.6	23.6	12.6
	(21.5)	(19.0)	(22.6)	(13.9)	(—)
Imports					
$\Delta\mathbf{L}$	61.5	59.5	55.2	44.5	28.9
	(48.4)	(47.7)	(38.4)	(17.6)	(—)
$\Delta\mathbf{B}$	67.1	56.5	43.2	38.6	—
	(97.1)	(44.6)	(17.0)	(11.0)	(—)
$\Delta\mathbf{f}$	37.6	30.5	29.3	18.3	4.9
	(21.5)	(19.0)	(22.6)	(13.9)	(—)

[a]The summary statistics for the $\Delta\hat{\mathbf{v}}$ effects for the imports decomposition are identical to those of the $\Delta\hat{\mathbf{u}}$ effects for the labour costs decomposition, so they are omitted. The figures denote the average of the absolute r_i/μ_i values (as a percentage). The terms in parentheses denote the standard deviation.
[b]Since the number of sectors equals one, there is only one observation. By definition, this implies that the standard deviation is zero, so it is omitted.
[c]At this aggregation level, by definition, the bridge matrix (which describes the distribution of the final demand components over the sectors) consists of ones, so does not change.

With aggregation before decomposition, as discussed already, the underlying IO data are aggregated first, after which the decomposition is applied. Another possibility is aggregation after decomposition, which occurs when the calculations are based on detailed IO data but are presented in an aggregate manner.[15] For example, the total labour costs are obtained by the summation of the sectoral labour costs, i.e.

$$\mathbf{W} = \Sigma_i w_i = \mathbf{e}'\mathbf{w}$$

where $\mathbf{e}'$ denotes the row summation vector $(1, \ldots, 1)$ of appropriate length. Similarly, $\mathbf{M} = \Sigma_i m_i = \mathbf{e}'\mathbf{m}$. From equation (12), it follows that the decomposition of the change in $\mathbf{W}$ is based on

$$\Delta\mathbf{W} = \mathbf{e}'\hat{\mathbf{u}}(92)\mathbf{L}(92)\mathbf{B}(92)\mathbf{f}(92) - \mathbf{e}'\hat{\mathbf{u}}(86)\mathbf{L}(86)\mathbf{B}(86)\mathbf{f}(86)$$

and again yields 24 different decomposition forms. The calculations of the four effects are obtained simply by summing the earlier outcomes over the sectors.

The results in Table 5 clearly show that aggregation after decomposition does not necessarily induce a drastic reduction in the variability. Intuitively speaking, one might be inclined to expect that, for some sectors, the outcomes obtained with a certain decomposition form are greater than the average outcome, while they are smaller for other sectors. Summing over the sectors would then imply that the differences from the average outcome would cancel each other out. As a consequence, one would expect that the r/μ results for the aggregate decomposition in Table 5, for example, would be much smaller than the average absolute r/μ values

Table 5. Results for aggregation after decomposition

	min	max	r	μ	σ	r/μ	σ/μ
Total labour costs, $\Delta \mathbf{W} = 79\,233$							
$\Delta \hat{\mathbf{u}}$	16 496	21 772	5 276	19 078	2 248	27.7	11.8
$\Delta \mathbf{L}$	3 907	5 258	1 351	4 557	489	29.6	10.7
$\Delta \mathbf{B}$	3 505	5 421	1 916	4 391	516	43.6	11.7
$\Delta \mathbf{f}$	47 850	54 486	6 636	51 208	2 803	13.0	5.5
Total imports, $\Delta \mathbf{M} = 25\,303$							
$\Delta \hat{\mathbf{v}}$	− 844	− 249	595	− 526	237	− 113.0	− 45.1
$\Delta \mathbf{L}$	− 1 737	− 1 097	640	− 1 408	265	− 45.5	− 18.8
$\Delta \mathbf{B}$	− 4 064	− 2 885	1 179	− 3 462	511	− 34.1	− 14.8
$\Delta \mathbf{f}$	30 026	31 331	1 305	30 699	548	4.3	1.2

reported in Table 3. It turns out that this only holds for the $\Delta \mathbf{f}$ effect in the decomposition of the change in the imports and, to a lesser extent, for the decomposition of the change in the labour costs. Observe that the variability of the $\Delta \hat{\mathbf{v}}$ effect even becomes much greater. This is caused by the fact that the average $\Delta \hat{\mathbf{v}}$ effect becomes relatively small after aggregation, while the range remains substantial.

4. Analyzing the *ad hoc* 'Solutions'

In this section, we analyze the two *ad hoc* 'solutions' to the non-uniqueness problem that have been used predominantly in the literature. The first solution is obtained by taking the average of the two polar decompositions in equations (5) and (6). The second solution is obtained by applying mid-point weights. In the simplest case, with only two determinants, both approaches yield the same result. In the general situation, with n determinants, this is no longer the case. Also, the attractive properties of the *ad hoc* solution in the simplest case (i.e. a simple weighting structure based on mid-point weights and the decomposition being exact) no longer hold in the general case. The average of the polar decompositions is still exact, but it does not exhibit a simple weighting structure. The decomposition using mid-point weights is no longer exact.

4.1. *All versus Polar Decompositions*

This subsection examines whether or not it is necessary to compute all the 24 (or $n!$ in general) decomposition forms in order to obtain some idea of the average effect and of the range of outcomes. As the alternative, we consider the average of the two polar decompositions in equations (13) and (14). For example, the average $\Delta \mathbf{L}$ effect from the polar decompositions yields

$$\Delta \mathbf{L} \text{ effect} = \tfrac{1}{2}\hat{\mathbf{u}}(86)(\Delta \mathbf{L})\mathbf{B}(92)\mathbf{f}(92) + \tfrac{1}{2}\hat{\mathbf{u}}(92)(\Delta \mathbf{L})\mathbf{B}(86)\mathbf{f}(86)$$

In contrast to the average effect from the polar decompositions, we have the average effect of the full set of all 24 decompositions.

Table 6. All versus polar decompositions

	Labour costs				Imports			
	$\Delta\hat{\mathbf{u}}$	$\Delta\mathbf{L}$	$\Delta\mathbf{B}$	$\Delta\mathbf{f}$	$\Delta\hat{\mathbf{v}}$	$\Delta\mathbf{L}$	$\Delta\mathbf{B}$	$\Delta\mathbf{f}$
$100(\mu_i^{\mathrm{P}}/\mu_i^{\mathrm{F}})$								
Average	99.97	99.59	99.81	99.80	99.97	100.95	101.12	99.98
Standard dev.	0.90	2.85	6.18	0.90	0.90	4.67	7.70	2.96
$\rho_i = 100(r_i^{\mathrm{P}}/r_i^{\mathrm{F}})$								
Average	69.62	50.14	50.42	61.91	69.41	69.49	68.20	65.00
Frequencies								
$0 \leqslant \rho_i < 50$	30.05	54.41	51.17	40.38	30.19	28.78	30.05	34.74
$50 \leqslant \rho_i < 100$	36.15	32.84	40.85	30.52	36.32	40.98	39.91	39.91
$\rho_i = 100$	33.80	12.75	7.98	29.11	33.49	30.24	30.05	25.35

For Table 6, for each sector i, we calculated the ratio (as a percentage) between the average effect as computed from the polar decompositions (i.e. μ_i^{P}) and the average effect as computed from the full set of decompositions (i.e. μ_i^{F}). Next, the average over all sectors and the standard deviation were computed. Similarly, ρ_i is defined as the ratio (as a percentage) of the two ranges, r_i^{P} and r_i^{F}. Again, the average over all sectors is calculated, while a small frequency table is also given, reporting the percentage of all sectors with a ρ_i value smaller than 50, larger than 50 or equal to 100.

The results show that, on average, the two average effects μ_i^{P} and μ_i^{F} are remarkably close to each other. The relatively small standard deviations indicate that this also holds for each sector separately. This suggests that, for estimating the average effects for the full decompositions (i.e. μ_i^{F}), it suffices to consider the average effects from the polar decompositions (i.e. μ_i^{P}).

The variability of the outcomes, however, is considerably underestimated when the polar decompositions are used instead of the full decompositions. Moreover, the magnitude of this underestimation may vary greatly from case to case. For example, for the $\Delta\mathbf{L}$ effect (resp. $\Delta\mathbf{B}$ effect) of the sectoral labour costs decomposition, the polar decompositions report a range (r_i^{P}) that is less than half the range for the full decompositions (i.e. r_i^{F}) for 54% (resp. 51%) of the sectors. For the $\Delta\mathbf{B}$ effect, we find that the same range is found for the polar and for the full decompositions (i.e. $r_i^{\mathrm{P}} = r_i^{\mathrm{F}}$) in only 8% of the sectors. In contrast, for the $\Delta\hat{\mathbf{u}}$ effect, the same range is found in 34% of the sectors. These results indicate that considering the polar decompositions instead of the full decompositions may be highly misleading, as far as the range (or standard deviation) is concerned. However, results in the rightmost four columns of Table 6, for the decomposition of the sectoral imports, show that this need not necessarily be the case. Comparing r_i^{P} with r_i^{F} for the effects of the import decomposition, we find that, for about 30% of the sectors, the range is more than halved, while also the range remains equal for 30% of the sectors.

4.2. Approximate Decompositions

In this subsection, we discuss the approach that uses the discrete approximation of the total differential (see, for example, Wolff, 1985, 1994; Afrasiabi & Casler, 1991). The *ad hoc* solution of applying mid-point weights is a special case.

Consider $\mathbf{w} = \hat{\mathbf{u}}\mathbf{LBf}$; then, we have

$$d\mathbf{w} = (d\hat{\mathbf{u}})\mathbf{LBf} + \hat{\mathbf{u}}(d\mathbf{L})\mathbf{Bf} + \hat{\mathbf{u}}\mathbf{L}(d\mathbf{B})\mathbf{f} + \hat{\mathbf{u}}\mathbf{LB}(d\mathbf{f}) \tag{15}$$

Using the discrete approximation yields

$$\Delta\mathbf{w} \approx (\Delta\hat{\mathbf{u}})\mathbf{LBf} + \hat{\mathbf{u}}(\Delta\mathbf{L})\mathbf{Bf} + \hat{\mathbf{u}}\mathbf{L}(\Delta\mathbf{B})\mathbf{f} + \hat{\mathbf{u}}\mathbf{LB}(\Delta\mathbf{f}) \tag{16}$$

In empirical studies, the calculation of this approximate decomposition is usually based on first-year weights, on last-year weights or on their average (i.e. mid-point weights). This reads as

$$\Delta\mathbf{w} = (\Delta\hat{\mathbf{u}})\mathbf{L}_0\mathbf{B}_0\mathbf{f}_0 + \hat{\mathbf{u}}_0(\Delta\mathbf{L})\mathbf{B}_0\mathbf{f}_0 + \hat{\mathbf{u}}_0\mathbf{L}_0(\Delta\mathbf{B})\mathbf{f}_0 + \hat{\mathbf{u}}_0\mathbf{L}_0\mathbf{B}_0(\Delta\mathbf{f}) + \varepsilon(0) \tag{17}$$

$$= (\Delta\hat{\mathbf{u}})\mathbf{L}_1\mathbf{B}_1\mathbf{f}_1 + \hat{\mathbf{u}}_1(\Delta\mathbf{L})\mathbf{B}_1\mathbf{f}_1 + \hat{\mathbf{u}}_1\mathbf{L}_1(\Delta\mathbf{B})\mathbf{f}_1 + \hat{\mathbf{u}}_1\mathbf{L}_1\mathbf{B}_1(\Delta\mathbf{f}) + \varepsilon(1) \tag{18}$$

$$= (\Delta\hat{\mathbf{u}})\mathbf{L}_{1/2}\mathbf{B}_{1/2}\mathbf{f}_{1/2} + \hat{\mathbf{u}}_{1/2}(\Delta\mathbf{L})\mathbf{B}_{1/2}\mathbf{f}_{1/2} + \hat{\mathbf{u}}_{1/2}\mathbf{L}_{1/2}(\Delta\mathbf{B})\mathbf{f}_{1/2} + \hat{\mathbf{u}}_{1/2}\mathbf{L}_{1/2}\mathbf{B}_{1/2}(\Delta\mathbf{f}) + \phi(\tfrac{1}{2}) \tag{19}$$

where, for example, $\varepsilon(0)$ covers the interaction effects. These are higher-order effects up to the order 4 (or n for the general decomposition). The averages are defined as before. Thus, for example, $\mathbf{L}_{1/2} = \frac{1}{2}\mathbf{L}_0 + \frac{1}{2}\mathbf{L}_1$. Note that $\phi(\frac{1}{2}) \neq \frac{1}{2}\varepsilon(0) + \frac{1}{2}\varepsilon(1)$, since, in general, $(\Delta\hat{\mathbf{u}})\mathbf{L}_{1/2}\mathbf{B}_{1/2}\mathbf{f}_{1/2} \neq \frac{1}{2}(\Delta\hat{\mathbf{u}})\mathbf{L}_0\mathbf{B}_0\mathbf{f}_0 + \frac{1}{2}(\Delta\hat{\mathbf{u}})\mathbf{L}_1\mathbf{B}_1\mathbf{f}_1$, etc. This implies that there is also a fourth alternative, i.e.

$$\begin{aligned}\Delta\mathbf{w} = {} & \tfrac{1}{2}(\Delta\hat{\mathbf{u}})\mathbf{L}_0\mathbf{B}_0\mathbf{f}_0 + \tfrac{1}{2}(\Delta\hat{\mathbf{u}})\mathbf{L}_1\mathbf{B}_1\mathbf{f}_1 + \tfrac{1}{2}\hat{\mathbf{u}}_0(\Delta\mathbf{L})\mathbf{B}_0\mathbf{f}_0 + \tfrac{1}{2}\hat{\mathbf{u}}_1(\Delta\mathbf{L})\mathbf{B}_1\mathbf{f}_1 \\ & + \tfrac{1}{2}\hat{\mathbf{u}}_0\mathbf{L}_0(\Delta\mathbf{B})\mathbf{f}_0 + \tfrac{1}{2}\hat{\mathbf{u}}_1\mathbf{L}_1(\Delta\mathbf{B})\mathbf{f}_1 + \tfrac{1}{2}\hat{\mathbf{u}}_0\mathbf{L}_0\mathbf{B}_0(\Delta\mathbf{f}) + \tfrac{1}{2}\hat{\mathbf{u}}_1\mathbf{L}_1\mathbf{B}_1(\Delta\mathbf{f}) + \varepsilon(\tfrac{1}{2})\end{aligned} \tag{20}$$

where $\varepsilon(\frac{1}{2}) = \frac{1}{2}\varepsilon(0) + \frac{1}{2}\varepsilon(1)$.[16]

Table 7 contains the results with respect to the variability of the outcomes of the three approximate forms in equations (17), (18) and (20), and summary statistics for the error terms $\varepsilon(.)$. For each sector i, the range r_i was obtained as the absolute difference of the $\Delta\mathbf{L}$ effect for example, in equation (18), and the $\Delta\mathbf{L}$ effect in equation (17). The average $\Delta\mathbf{L}$ effect μ_i was taken from equation (20). For each sector, the absolute r_i/μ_i value was calculated (as a percentage). The averages and standard deviations are reported in Table 7 for each of the four

Table 7. Approximate decompositions

	Labour costs		Imports	
	Average	Standard deviation	Average	Standard deviation
$100(r_i/\mu_i)$				
$\Delta\hat{\mathbf{t}}$[a]	28.5	17.5	28.4	17.6
$\Delta\mathbf{L}$	39.6	33.8	36.6	38.9
$\Delta\mathbf{B}$	52.7	185.7	44.2	114.3
$\Delta\mathbf{f}$	17.1	16.3	24.7	27.1
$100[\varepsilon_i(.)/\Delta T_i]$[b]				
$\varepsilon(0)$	25.4	76.3	59.3	187.3
$\varepsilon(1)$	23.1	67.6	55.6	166.3
$\varepsilon(\frac{1}{2})$	1.5	4.7	2.9	11.5

[a] $\Delta\hat{\mathbf{t}}$ is $\Delta\hat{\mathbf{u}}$ in the case of the labour costs, and $\Delta\hat{\mathbf{v}}$ in the case of the imports.
[b] ΔT_i is Δw_i in the case of the labour costs, and Δm_i in the case of the imports.

effects. The sectoral errors were obtained as a percentage of the total sectoral effect, i.e. $100\varepsilon_i(.)/\Delta w_i$ in the case of decomposing the labour costs. Table 7 reports the averages and the standard deviations of the absolute errors.

The results indicate large differences between the effects computed with first-year weights and those computed with last-year weights. Also, the errors are substantial, although the reported averages are somewhat suggestive. It should be mentioned that the weighted averages, using $|\Delta w_i|$ as weights, of the absolute sectoral error percentages are substantially smaller. For the labour costs decomposition, we find weighted averages of 11.2% for $\varepsilon(0)$, 11.7% for $\varepsilon(1)$ and 0.4% for $\varepsilon(\frac{1}{2})$. For the import decomposition, the corresponding figures are 18.7%, 19.1% and 0.9%. These differences between the weighted averages and the averages reported in Table 7 indicate that the larger absolute error percentages are observed for sectors with relatively small total effects Δw_i and Δm_i. In contrast to the cases with first-year and last-year weights, the approximation in equation (20) performs well. Most sectors have very small absolute errors. For example, for the decomposition of the change in the labour costs, there are 211 sectors for which $\Delta w_i \neq 0$. Of these, 179 have an absolute error smaller than 2%, only five have an absolute error greater than 10% and the largest reported error was 58.4%.

Instead of using the average of the effects with first-year and last-year weights, one may use the effects obtained with mid-point weights, i.e. using equation (19) instead of equation (20). Table 8 analyzes the differences between these two alternatives. To this end, the ratio between the effects computed with equation (19) and those computed with equation (20) is calculated for each sector. It turns out that the results are very similar, in the sense that the ratios are very close to unity. In Table 8, this is reflected by an average ratio that is approximately 100 with a small standard deviation.[17]

Table 8 also shows that the approximate decomposition of equation (19) is almost exact. For the labour costs decomposition, the average absolute error is less than 1%; the weighted average is even only 0.2%. For the 211 sectors with $\Delta w_i \neq 0$, no less than 197 have an absolute error $\phi(\frac{1}{2})$ smaller than 2%, while only two sectors show an absolute error larger than 10%. The findings for the decomposition of the changes in the imports sketch a similar picture.

The central problem with the application of decomposition techniques is finding appropriate weights. For the exact decompositions in Sections 2 and 3, each effect is weighted in a different manner. For example, in equation (13), the $\Delta\hat{\mathbf{u}}$ effect has

Table 8. Approximate decomposition with mid-point weights

	Labour costs		Imports	
	Average	Standard deviation	Average	Standard deviation
$100[\mu_i^{(19)}/\mu_i^{(20)}]$				
$\Delta\hat{\mathbf{t}}^a$	100.05	1.37	100.06	1.37
$\Delta\mathbf{L}$	99.56	3.20	100.60	3.19
$\Delta\mathbf{B}$	101.03	16.00	101.24	7.66
$\Delta\mathbf{f}$	100.31	1.35	100.12	3.48
$100[\phi_i(.)/\Delta T_i]^b$	0.75	2.36	1.45	5.75

[a] $\Delta\hat{\mathbf{t}}$ is $\Delta\hat{\mathbf{u}}$ in the case of the labour costs, and $\Delta\hat{\mathbf{v}}$ in the case of the imports.
[b] ΔT_i is Δw_i in the case of the labour costs, and Δm_i in the case of the imports.

a different *type* of weight compared with the $\Delta\mathbf{f}$ effect. Moreover, for exact decompositions, in general, $n!$ equivalent forms exist. Approximate decompositions seem to solve these problems, but only at first sight. The results in Table 7 show that using either first-year or last-year weights causes serious errors. The errors vanish almost completely, however, when the average effects are considered or when the effects are computed using mid-point weights. Intuitively speaking, this result is most appealing, since the approximate decomposition in equation (19) is almost exact and has the attractive feature that the same types of weight are used for each effect. It should be borne in mind, however, that using the decomposition in equation (19) only conceals the problem of a large variation in the outcomes of the exact decompositions, as indicated by the empirical analysis in the previous section. At best, therefore, the approximate decomposition in equation (19) may be viewed as being an additional alternative to the existing 24 (or $n!$ in general) equivalent exact decompositions.

5. Conclusions

In this paper, we have addressed the problem that there is no unique form to decompose the change in one variable (or vector, or matrix of variables) into the changes in its determinants. In the simplest case, with only two determinants, the problem is usually solved on an *ad hoc* basis by taking the average of the two possible forms. The resulting decomposition has the intuitive advantage that mid-point (or average) weights are used for the change in each of the determinants. In our view, this simple and intuitive solution has distracted the attention from the potential seriousness of the non-uniqueness problem. This is because, in the general case with n determinants, there are $n!$ equivalent decomposition forms, and a single *ad hoc* solution with the same 'nice' properties as in the simplest case no longer exists.

In evaluating the extent of the non-uniqueness problem, an empirical analysis was carried out for The Netherlands based on the 214-sector IO tables for 1986 and 1992. The changes in sectoral labour costs and the sectoral imports were decomposed into four underlying sources. Results were calculated for all $4! = 24$ decomposition forms. The outcomes exhibit considerable variability, so the contribution of a certain source appears to depend crucially on the way that it is measured. Since structural decomposition analysis aims to quantify the contribution of each underlying course, the sensitivity with respect to the chosen decomposition form casts doubts on its sense. The major finding of considerable variability is not affected by the number of sectors taken into account. The decomposition results after the original data have been aggregated only show a slowly declining variability.

Next, we have analyzed the two *ad hoc* solutions to the non-uniqueness problem that have been used predominantly in the literature: taking the average of the results of the two polar decompositions and applying mid-point weights. Recall that, in the simplest case of only two determinants, both approaches are the same. When there are n determinants, the average of the two polar decompositions does not yield a form with a simple, intuitively appealing weighting structure, unless $n = 2$. Using mid-point weights only yields an approximate decomposition (i.e. where the sum of all contributions does not sum to 100%, in general), again unless $n = 2$.

Comparing the average of the two polar decompositions with the average of the full set of 24 decompositions showed that the results are extremely close to each other, even at the sectoral level. Analyzing the range of the two polar decompositions versus the range of all the decompositions gave very diverse results. For one of the four sources, we found that, in 34% of the sectors, both ranges were equal to each other, while, in 30% of the sectors, the ratio between the two ranges was less than 0.5. For another source, however, the corresponding percentages were 8% and 51%.

The results for the approximate decompositions (which are also non-unique) showed that, when mid-point weights are applied, the approximation is almost exact. However, all that this says is that, at best, using mid-point weights provides another alternative to the existing $n!$ exact decompositions.

Applying a structural decomposition technique can be carried out with a multitude of computational forms which are different but which are equivalent in the sense that no form is to be preferred on theoretical grounds to the others. Since the *ad hoc* solutions to this non-uniqueness problem that have been used predominantly appear to be unsatisfactory for cases with more than two determinants, we suggest that the average should be computed. However, given the fact that the results may differ greatly between the different forms, we feel that the range (or the standard deviation) also provides relevant information. Therefore, we would recommend that, in empirical analyses, the average effects as well as the ranges (or the standard deviations) are published.

Notes

1. This is an adaptation of the first formal definition by Rose and Miernyk (1989, p. 245).
2. Dollar and Wolff (1988, 1993) and Bernard and Jones (1996) go one step further, and decompose the convergence of aggregate labour productivities and aggregate total factor productivities of several countries into productivity convergence at the sectoral level and productivity effects caused by shifts in the employment or the output mixes.
3. For example, Fromm (1968) provides a link with the index number problem (see also Schumann, 1994).
4. Alternatives that include interaction terms are $\Delta y = (\Delta x)z(1) + x(1)(\Delta z) - (\Delta x)(\Delta z)$ or $\Delta y = (\Delta x)z(0) + x(0)(\Delta z) + (\Delta x)(\Delta z)$. Note also that the mean of these two expressions yields equation (3).
5. It should be mentioned that these $n!$ decomposition forms do not exhaust the possibilities. For example, use $\Delta y = [\Delta(x_1 \ldots x_i)][(x_{i+1} \ldots x_n)(1)] + [(x_1 \ldots x_i)(0)][\Delta(x_{i+1} \ldots x_n)]$ for $i = 1, \ldots, n$ and decompose the Δ terms further along the same lines. This provides a set of decomposition forms with a different structure (see, for example, Dietzenbacher and Los, 1997).
6. This holds in particular, since the non-uniqueness problem has been extensively dealt with, both theoretically and empirically, in the literature on index numbers. In the field of IO analysis, an exception is Dietzenbacher and Los (1997), who examine the decomposition of changes in the aggregated Leontief inverse and in the value added per sector. Using highly aggregated data, the analysis of the variability is based purely on the arithmetical equivalence of decomposition forms, implying that the majority of forms have inconsistent structures. As a consequence, some forms are to be preferred to others. In contrast, the present decompositions are based on economic equivalance where no single form is to be preferred over the others.
7. $\hat{\mathbf{u}}$ denotes the diagonal matrix with the vector $\mathbf{u}$ on its main diagonal and all other entries equal to zero.
8. In the empirical analysis, we have used IO tables in current prices. Although it is generally preferable to use tables in constant prices, the present data set suffices for the paper's purpose to analyze the sensitivity of the results with respect to the chosen decomposition form. It should be emphasized, however, that $\Delta\mathbf{A}$ not only describes technical changes but also covers price changes. Strictly speaking, $\Delta\mathbf{L}$ then gives the effects of a changing cost structure.
9. This change may be decomposed even further into the change in the overall final demand level and the change in the distribution of the overall final demand over the five categories (see, for example, Lin & Polenske, 1995).
10. Although the data have not been published in printed form, they are available on diskette from

Statistics Netherlands. For general information or details on the sector classification, see, for example, Statistics Netherlands (1996).

11. Note that, for each sector, the four figures in the column under μ_i add up to the reported change Δw_i (for example, 897 for sector 156).
12. For example, the average is, in principle, determined as $\Sigma_{i=1}^{214}|100(r_i/\mu_i)|/214$. Some sectors, however, were not included, because $r_i = \sigma_i = \mu_i = 0$.
13. Observe that the reported results for the $\Delta\hat{\mathbf{u}}$ effect are identical to those for the $\Delta\hat{\mathbf{v}}$ effect. Although the effects differ from each other, it is easily shown that their r_i/μ_i and σ_i/μ_i ratios are equal to each other, for each sector.
14. Although a classification of 20–30 sectors is the highest level of aggregation that still bears some economic relevance for applied analyses, the one-sector case is included for completeness.
15. In a similar fashion, Wolff (1985, 1994) decomposes the change in the overall rate of total factor productivity growth into a value share effect, an inter-industry effect reflected by the change in the Leontief inverse matrix and a sectoral technical change effect.
16. Note that, for a decomposition in two components (say $x = yz$), we have $\varepsilon(0) = (\Delta x)(\Delta z)$, $\varepsilon(1) = -(\Delta x)(\Delta z)$ and $\varepsilon(\frac{1}{2}) = 0$. For the case of two components, the approximate decomposition with average weights is exact and equal to equation (3). This does not hold in general, however.
17. The relatively large standard deviation for the $\Delta\mathbf{B}$ effect in the decomposition of the change in the labour costs is caused by one sector with very small $\Delta\mathbf{B}$ effects, both with equation (19) and equation (20).

References

Afrasiabi, A. & Casler, S. D. (1991) Product-mix and technological change within the Leontief inverse, *Journal of Regional Science*, 31, pp. 147–160.

Barker, T. (1990) Sources of structural change for the UK service industries 1979–84, *Economic Systems Research*, 2, pp. 173–183.

Bernard, A. B. & Jones, C. I. (1996) Productivity across industries and countries: time series theory and evidence, *Review of Economics and Statistics*, 78, pp. 135–146.

Carter, A. P. (1970) *Structural Change in the American Economy* (Cambridge, MA, Harvard University Press).

Chenery, H. B., Shishido, S. & Watanabe, T. (1962) The pattern of Japanese growth, 1914–1954, *Econometrica*, 30, pp. 98–139.

Denison, E. F. (1974) *Accounting for United States Economic Growth 1929–1969* (Washington, DC, The Brookings Institution).

Denison, E. F. (1985) *Trends in American Economic Growth 1929–1982* (Washington, DC, The Brookings Institution).

Dietzenbacher, E. & Los, B. (1997) Analyzing Decomposition Analyses, in: A. Simonovits & A. E. Steenge (eds) *Prices, Growth and Cycles*, pp. 108–131 (London, Macmillan).

Dollar, D. & Wolff, E. N. (1988) Convergence of industry labor productivity among advanced economies, 1963–82, *Review of Economics and Statistics*, 70, pp. 549–558.

Dollar, D. & Wolff, E. N. (1993) *Competitiveness, Convergence, and International Specialization* (Cambridge, MA, MIT Press).

Feldman, S. J., McClain, D. & Palmer, K. (1987) Sources of structural change in the United States, 1963–78: an input–output perspective, *Review of Economics and Statistics*, 69, pp. 503–510.

Fontela, E. (1989) Industrial structures and economic growth: an input–output perspective, *Economic Systems Research*, 1, pp. 45–52.

Forssell, O. (1990) The input–output framework for analysing changes in the use of labour by education levels, *Economic Systems Research*, 2, pp. 363–376.

Fromm, G. (1968) 'Comment' on Vaccara and Simon, in: J. W. Kendrick (ed.) *The Industrial Composition of Income and Product*, pp. 59–66 (New York, Columbia University Press).

Fujimagari, D. (1989) The sources of change in Canadian industry output, *Economic Systems Research*, 1, pp. 187–201.

Galatin, M. (1988) Technical change and the measurement of productivity in an input–output model, *Journal of Macroeconomics*, 10, pp. 613–632.

Kanemitsu, H. & Ohnishi, H. (1989) An input–output analysis of technological changes in the Japanese economy: 1970–1980, in: R. E. Miller, K. R. Polenske & A. Z. Rose (eds) *Frontiers of Input–Output Analysis*, pp. 308–323 (New York, Oxford University Press).

Kendrick, J. W. (1961) *Productivity Trends in the United States* (Princeton, NJ, Princeton University Press).

Leontief, W. (1953) Structural change, in: W. Leontief *et al.* (eds) *Studies in the Structure of the American Economy*, pp. 17–52 (New York, Oxford University Press).

Leontief, W. & Ford, D. (1972) Air pollution and the economic structure: empirical results of input–output computations, in: A. Bródy & A. P. Carter (eds) *Input–Output Techniques*, pp. 9–30 (Amsterdam, North-Holland).

Lin, X. & Polenske, K. R. (1995) Input–output anatomy of China's energy use changes in the 1980s, *Economic Systems Research*, 7, pp. 67–84.

Oosterhaven, J. (1981) *Interregional Input–Output Analysis and Dutch Regional Policy Problems* (Aldershot, Gower).

Oosterhaven, J., Hoen, A. R. & van der Linden, J. A. (1995) Technology, trade and real value added growth of EC countries, 1975–1985, paper presented at the *11th International Conference on Input–Output Techniques, New Delhi.*

Rose, A. & Casler, S. (1996) Input–output structural decomposition analysis: a critical appraisal, *Economic Systems Research*, 8, pp. 33–62.

Rose, A. & Miernyk, W. (1989) Input–output analysis: the first fifty years, *Economic Systems Research*, 1, pp. 229–271.

Schumann, J. (1994) Does it make sense to apply the static open input–output model for imputation and structural decomposition?, *Economic Systems Research*, 6, pp. 171–178.

Skolka, J. (1989) Input–output structural decomposition analysis for Austria, *Journal of Policy Modeling*, 11, pp. 45–66.

Solow, R. M. (1957) Technical change and the aggregate production function, *Review of Economics and Statistics*, 39, pp. 312–320.

Stäglin, R. & Wessels, H. (1972) Intertemporal analysis of structural change in the German economy, in: A. Bródy & A. P. Carter (eds) *Input–Output Techniques*, pp. 370–392 (Amsterdam, North-Holland).

Statistics Netherlands (1995) *National Accounts 1994* (The Hague, SDU Publishers) (in Dutch).

Statistics Netherlands (1996) *National Accounts of the Netherlands 1995* (The Hague, SDU Publishers).

Vaccara, B. N. & Simon, N. W. (1968) Factors affecting postwar industrial composition of real product, in: J. W. Kendrick (ed) *The Industrial Composition of Income and Product*, pp. 19–58 (New York, Columbia University Press).

Wolff, E. N. (1985) Industrial composition, interindustry effects, and the U.S. productivity slowdown, *Review of Economics and Statistics*, 67, pp. 268–277.

Wolff, E. N. (1994) Productivity measurement within an input–output framework, *Regional Science and Urban Economics*, 24, pp. 75–92.

Wolff, E. N. & Howell, D. R. (1989) Labor quality and productivity growth in the United States: an input–output growth-accounting framework, in: R. E. Miller, K. R. Polenske & A. Z. Rose (eds) *Frontiers of Input–Output Analysis*, pp. 148–162 (New York, Oxford University Press).