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On the Problem of Sensor Placement

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Abstract

In this paper we consider an approach to solve the problem of sensor placement. This approach is based on constructing logical models for the problem.

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Different formalizations of the problem of sensor placement received a lot of attention recently (see e.g. [1, 2]). For instance, sensor placement is extensively used for improved robotic navigation (see e.g. [3, 4]). In particular, visual landmarks problems are extensively studied in contemporary robotics (see e.g. [5, 6, 7]). In this paper we consider SP problem (see [2]).

Let

$$[c]_2 \rightleftharpoons c\langle 1\rangle c\langle 2\rangle \ldots c\langle \lceil \log_2 m\rceil\rangle$$

where

$$c = \sum_{i=1}^{\lceil \log_2 m\rceil} 2^{\lceil \log_2 m\rceil - i}\langle i\rangle,$$

$c \leq m$. Let

$$\delta_i = \wedge_{1\leq c\leq m}((\wedge_{1\leq j\leq \lceil \log_2 m\rceil} y_{i,j} = c\langle j\rangle) \rightarrow$$

$$((\wedge_{l\in\{p|b_p\in F(a_c)\}} z_{i,l}=1)\wedge(\wedge_{l\in\{p|b_p\notin F(a_c)\}} z_{i,l}=0))),$$

$$\varepsilon=\wedge_{1\le l\le n}(\vee_{1\le i\le k} z_{i,l}),$$

$$\zeta_i=\neg(\wedge_{1\le c\le m}(\vee_{1\le j\le\lceil\log_2 m\rceil} y_{i,j}\ne c\langle j\rangle)),$$

$$\psi=(\wedge_{1\le i\le k}\delta_i)\wedge\varepsilon\wedge(\wedge_{1\le i\le k}\zeta_i)$$

where $1\le i\le k$.

Theorem. *There is $T\subseteq S$ such that $\cup_{x\in T}F(x)=N$ and $|T|\le k$ if and only if ψ is satisfiable.*

Proof. Suppose that there is $T\subseteq S$ such that $\cup_{x\in T}F(x)=N$ and $|T|\le k$. Without loss of generality we can assume that $|T|=k$.

Let $T=\{c_1,c_2,\ldots,c_k\}$. Let $y_{i,j}=i\langle j\rangle$ where $1\le i\le k$, $z_{i,l}=1$ for $l\in\{p\mid b_p\in F(c_i)\}$, $z_{i,l}=0$ for $l\in\{p\mid b_p\notin F(c_i)\}$. Satisfiability of δ_i and ζ_i follows directly from the choice of values of variables.

Since $\cup_{x\in T}F(x)=N$, for any $b_p\in N$ there is $c_i\in T$ such that $b_p\in F(c_i)$. Thus, for any p, $1\le p\le n$, there is i such that $z_{i,p}=1$. So, ε is satisfiable. Therefore, ψ is satisfiable.

Suppose now that ψ is satisfiable. Consider some assignment to the variables of ψ such that ψ is satisfiable. Since ψ is satisfiable, it is easy to see that ε is satisfiable. Thus, for any l, $1\le l\le n$, there is i, $1\le i\le k$, such that $z_{i,l}=1$. Since ζ_i is satisfiable, $1\le i\le k$, it is easy to check that for any i and some c, $1\le c\le m$, we have $c\langle j\rangle y_{i,j}$ where $1\le j\le\lceil\log_2 m\rceil$. Thus, in view of $z_{i,l}=1$, we have $l\in\{p\mid b_p\in F(a_c)\}$. Therefore, for any l, $1\le l\le n$, there are values of $y_{i,1},y_{i,2},\ldots,y_{i,\lceil\log_2 m\rceil}$ such that there is c, $1\le c\le m$, such that $c\langle j\rangle y_{i,j}$ where $1\le j\le\lceil\log_2 m\rceil$ and $l\in\{p\mid b_p\in F(a_c)\}$. By definition of ψ, there are no more then k different assignments for $y_{i,1},y_{i,2},\ldots,y_{i,\lceil\log_2 m\rceil}$. Therefore, there is $T\subseteq S$ such that $\cup_{x\in T}F(x)=N$ and $|T|\le k$. □

It is easy to see that

$$\zeta_i\Leftrightarrow\zeta_i'=\wedge_{m<c\le 2^{\lceil\log_2 m\rceil}}(\vee_{1\le j\le\lceil\log_2 m\rceil} y_{i,j}\ne c\langle j\rangle).$$

It is clear that

$$\delta_i\Leftrightarrow\wedge_{1\le c\le m}((\vee_{1\le j\le\lceil\log_2 m\rceil} y_{i,j}\ne c\langle j\rangle)\vee$$
$$((\wedge_{l\in\{p|b_p\in F(a_c)\}} z_{i,l}=1)\wedge(\wedge_{l\in\{p|b_p\notin F(a_c)\}} z_{i,l}=0))).$$

Therefore,

$$\delta_i\Leftrightarrow\delta_i'=\wedge_{1\le c\le m}(((\wedge_{l\in\{p|b_p\in F(a_c)\}}(z_{i,l}=1\vee$$
$$(\vee_{1\le j\le\lceil\log_2 m\rceil} y_{i,j}\ne c\langle j\rangle)))\wedge$$
$$(\wedge_{l\in\{p|b_p\notin F(a_c)\}}(z_{i,l}=0\vee(\vee_{1\le j\le\lceil\log_2 m\rceil} y_{i,j}\ne c\langle j\rangle))))).$$

Thus,

$$\psi\Leftrightarrow\psi'=(\wedge_{1\le i\le k}\delta_i')\wedge\varepsilon\wedge(\wedge_{1\le i\le k}\zeta_i').$$

In view of

$$x = 1 \Leftrightarrow x, \qquad x \neq 1 \Leftrightarrow \neg x,$$
$$x = 0 \Leftrightarrow \neg x, \qquad x \neq 0 \Leftrightarrow x,$$

it is clear that ψ' is a CNF. It is easy to check that ψ' gives us an explicit reduction from SP to SAT.

By direct verification we can check that

$$\begin{aligned}
\alpha \quad &\Leftrightarrow \quad (\alpha \vee \beta_1 \vee \beta_2) \wedge \\
&\quad\quad (\alpha \vee \neg\beta_1 \vee \beta_2) \wedge \\
&\quad\quad (\alpha \vee \beta_1 \vee \neg\beta_2) \wedge \\
&\quad\quad (\alpha \vee \neg\beta_1 \vee \neg\beta_2), \qquad (1)\\
\vee_{j=1}^{l}\alpha_j \quad &\Leftrightarrow \quad (\alpha_1 \vee \alpha_2 \vee \beta_1) \wedge \\
&\quad\quad (\wedge_{i=1}^{l-4}(\neg\beta_i \vee \alpha_{i+2} \vee \beta_{i+1})) \wedge \\
&\quad\quad (\neg\beta_{l-3} \vee \alpha_{l-1} \vee \alpha_l), \qquad (2)\\
\alpha_1 \vee \alpha_2 \quad &\Leftrightarrow \quad (\alpha_1 \vee \alpha_2 \vee \beta) \wedge \\
&\quad\quad (\alpha_1 \vee \alpha_2 \vee \neg\beta), \qquad (3)\\
\vee_{j=1}^{4}\alpha_j \quad &\Leftrightarrow \quad (\alpha_1 \vee \alpha_2 \vee \beta_1) \wedge \\
&\quad\quad (\neg\beta_1 \vee \alpha_3 \vee \alpha_4) \qquad (4)
\end{aligned}$$

where $l > 4$. Using relations (1) – (4) we can easily obtain an explicit transformation ψ' into ψ'' such that $\psi' \Leftrightarrow \psi''$ and ψ'' is a 3-CNF. It is clear that ψ'' gives us an explicit reduction from SP to 3SAT.

In papers [8, 9, 10, 11, 12] the authors considered some algorithms to solve logical models (see also [13, 14, 15, 16]). Our computational experiments have shown that these algorithms can be used to solve logical models for SP.

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