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THE THEORY OF LOCAL BEABLES

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A B S T R A C T

An attempt is made to formulate more explicitly a notion of "local causality" : correlations between physical events in different space-time regions should be explicable in terms of physical events in the overlap of the backward light cones. It is shown that ordinary relativistic quantum field theory is not locally causal in this sense, and cannot be embedded in a locally causal theory.

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INTRODUCTION : THE THEORY OF LOCAL BEABLES

This is a pretentious name for a theory which hardly exists otherwise, but which ought to exist. The name is deliberately modelled on "the algebra of local observables". The terminology, be-able as against observ-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory. For, in the words of Bohr [1], "it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms". It is the ambition of the theory of local beables to bring these "classical terms" into the mathematics, and not relegate them entirely to the surrounding talk.

The concept of "observable" lends itself to very precise mathematics when identified with "self-adjoint operator". But physically, it is a rather wooly concept. It is not easy to identify precisely which physical processes are to be given the status of "observations" and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described in "classical terms", because they are there. The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments. "Observables" must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables.

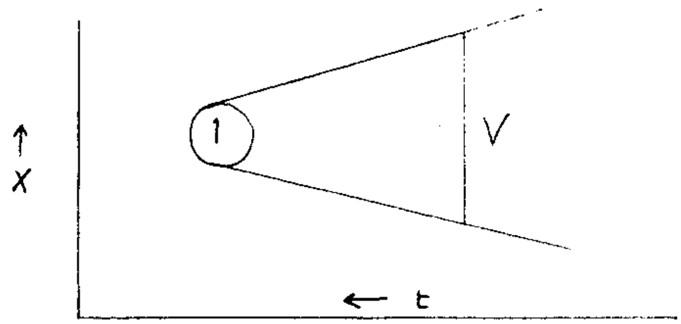
The word "beable" will also be used here to carry another distinction, that familiar already in classical theory between "physical" and "non-physical" quantities. In Maxwell's electromagnetic theory, for example, the fields $\vec{E}$ and $\vec{H}$ are "physical" (beables, we will say) but the potentials $\vec{A}$ and ϕ are "non-physical". Because of gauge invariance the same physical situation can be described by very different potentials. It does not matter that in Coulomb gauge the scalar potential propagates with infinite velocity. It is not really supposed to be there. It is just a mathematical convenience.

One of the apparent non-localities of quantum mechanics is the instantaneous, over all space, "collapse of the wave function" on "measurement". But this does not bother us if we do not grant beable status to the wave function. We can regard it simply as a convenient but inessential mathematical device for formulating correlations between experimental procedures and experimental results, i.e., between one set of beables and another. Then its odd behaviour is as acceptable as the funny behaviour of the scalar potential of Maxwell's theory in Coulomb gauge.

We will be particularly concerned with <u>local</u> beables, those which (unlike for example the total energy) can be assigned to some bounded space time region. For example, in Maxwell's theory the beables local to a given region are just the fields $\vec{E}$ and $\vec{H}$, in that region, and all functionals thereof. It is in terms of local beables that we can hope to formulate some notion of local causality. Of course we may be obliged to develop theories in which there <u>are</u> no strictly local beables. That possibility will not be considered here.

1) <u>Local determinism</u>

In Maxwell's theory, the fields in any space-time region 1 are determined by those in any space region V, at some time t, which fully closes the backward light cone of 1 :



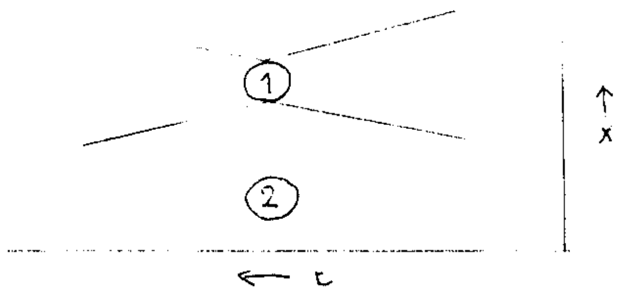
Because the region V is limited, localized, we will say the theory exhibits <u>local determinism</u>. We would like to form some notion of <u>local causality</u> in theories which are not deterministic, in which the correlations prescribed by the theory, for the beables, are weaker.

2) <u>Local causality</u>

Consider a theory in which the assignment of values to some beables Λ implies, not necessarily a particular value, but a probability distribution, for another beable A. Let

$$\{ A \mid \Lambda \}$$

denote the probability of a particular value A given particular values Λ. Let A be localized in a space-time region 1. Let B be a second beable localized in a second region 2 separated from 1 in a spacelike way



Now my intuitive notion of local causality is that events in 2 should not be "causes" of events in 1, and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the overlap of their backward light cones. It is perfectly intelligible then that if Λ in (1) does not contain a complete record of events in that overlap, it can be usefully supplemented by information from region 2. So in general it is expected that

$$\{ A \mid \Lambda, B \} \neq \{ A \mid \Lambda \} \tag{1}$$

However, in the particular case that Λ contains already a <u>complete</u> specification of beables in the overlap of the two light cones, supplementary information from region 2 could reasonably be expected to be redundant. So, with some change of notation, we formulate local causality as follows.

Let N denote a specification of <u>all</u> the beables, of some theory, belonging to the overlap of the backward light cones of spacelike separated regions 1 and 2. Let Λ be a specification of some beables from the remainder of the backward light cone of 1, and B of some beables in the region 2. Then in a <u>locally causal theory</u>

$$\{ A \mid \Lambda, N, B \} = \{ A \mid \Lambda, N \} \tag{2}$$

whenever both probabilities are given by the theory.

3) <u>Quantum mechanics is not locally causal</u>

Ordinary quantum mechanics, even the relativistic quantum field theory, is not locally causal in the sense of (2). Suppose, for example, we have a radioactive nucleus which can emit a single α particle, surrounded at a considerable distance by α particle counters. So long as it is not specified that some <u>other</u> counter registers, there is a chance for a particular counter that it registers. But if it is specified that some other counter does register, even in a region of space-time outside the relevant backward light cone, the chance that the given counter registers is zero. We simply do not have (2). Could it be that here we have an incomplete specification of the beables N ? Not so long as we stick to the list of beables recognized in ordinary quantum mechanics - the settings of switches and knobs and currents needed to prepare the initial unstable nucleus. For these are completely summarized, in so far as they are relevant for predictions about counter registering, in so far as such predictions are possible in quantum mechanics, by the wave function.

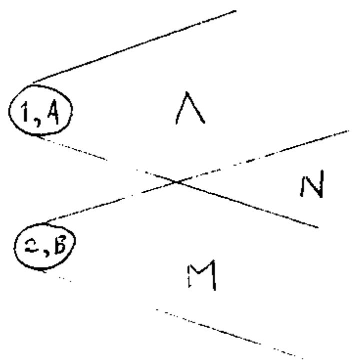
But could it not be that quantum mechanics is a fragment of a more complete theory, in which there are other ways of using the given beables, or in which there are additional beables - hitherto "hidden" beables ? And could it not be that this more complete theory has local causality ? Quantum mechanical predictions would then apply not to given values of all the beables, but to some probability distribution over them, in which the beables recognized as relevant by quantum mechanics are held fixed. We will investigate this question, and answer it in the negative.

4) Locality inequality [2)-25)]

Consider a pair of beables A and B, belonging respectively to regions 1 and 2 with spacelike separation, which happen by definition to have the property

$$|A| \leq 1 \qquad |B| \leq 1 \tag{3}$$

Consider the situation in which beables Λ, M, N are specified, where N is a complete specification of the beables in the overlap of the light cones, and Λ and M belong respectively to the remainders of the two light cones



Consider the joint probability distribution

$$\{A, B \mid \Lambda, M, N\} \tag{4}$$

By a standard rule of probability, it is equal to

$$\{A \mid \Lambda, M, N, B\}\{B \mid \Lambda, M, N\} \tag{5}$$

which, by (2), is the same as

$$\{A \mid \Lambda, N\}\{B \mid M, N\} \tag{6}$$

This says simply that correlations between A and B can arise only because of common causes N.

Consider now the expectation value of the product AB

$$p(\Lambda, M, N) = \sum_{A,B} AB\{A \mid \Lambda, N\}\{B \mid M, N\} \tag{7}$$

(where the summation stands also, if necessary, for integration)

$$= \bar{A}(\Lambda, N)\ \bar{B}(M, N) \tag{8}$$

where $\bar{A}$ and $\bar{B}$ are functions of the variables indicated, and

$$|\bar{A}| \leqslant 1 \qquad |\bar{B}| \leqslant 1 \tag{9}$$

for all values of the arguments. Let Λ' and M' be alternative specifications, of the same regions, to Λ and M.

$$\begin{aligned} p(\Lambda, M, N) \pm p(\Lambda, M', N) &= \bar{A}(\Lambda, N)\left[\bar{B}(M, N) \pm \bar{B}(M', N)\right] \\ p(\Lambda', M, N) \pm p(\Lambda', M', N) &= \bar{A}(\Lambda', N)\left[\bar{B}(M, N) \pm \bar{B}(M', N)\right] \end{aligned} \tag{10}$$

whence, using (9),

$$\begin{aligned} |p(\Lambda, M, N) \pm p(\Lambda, M', N)| &\leqslant |\bar{B}(M, N) \pm \bar{B}(M', N)| \\ |p(\Lambda', M, N) \pm p(\Lambda', M', N)| &\leqslant |\bar{B}(M, N) \pm \bar{B}(M', N)| \end{aligned} \tag{11}$$

so that finally, again invoking (9),

$$\left|p(\Lambda,M,N) \pm p(\Lambda,M',N)\right| + \left|p(\Lambda',M,N) \mp p(\Lambda',M',N)\right| \leq 2 \qquad (12)$$

Suppose now the specifications Λ, M, N are each given in two parts

$$\Lambda \equiv (a, \lambda)$$
$$M \equiv (b, \mu)$$
$$N \equiv (c, \nu)$$

where we are particularly interested in the dependence on a, b, c, while λ, μ, ν, are averaged over some probability distributions - which may depend on a, b, c. In the comparison with quantum mechanics, we will think of a, b, c, as variables which specify the experimental set-up in the sense of quantum mechanics, while λ, μ, ν, are in that sense either hidden or irrelevant. Define

$$P(a, b, c) = \overline{p((a,\lambda),(b,\mu),(c,\nu))} \qquad (13)$$

where the bar denotes the averaging over (λ,μ,ν) just described. Now applying again the locality hypothesis (3), the distribution of λ and ν must be independent of b, μ - the latter being outside the relevant backward light cones. So

$$\left|P(a,b,c) \pm P(a,b',c)\right| \leq \overline{\left|p((a,\lambda),(b,\mu),(c,\nu)) \pm p((a,\lambda),(b',\mu'),(c,\nu))\right|} \qquad (14)$$

- because the mod of the average is less than the average of the mod. In the same way

$$\left|P(a',b,c) \mp P(a',b',c)\right| \leq \overline{\left|p((a',\lambda'),(b,\mu),(c,\nu)) \mp p((a',\lambda'),(b',\mu'),(c,\nu))\right|} \qquad (15)$$

Finally then, from (14), (15) and (12),

$$\left|P(a,b,c) \mp P(a,b',c)\right| + \left|P(a',b,c) \pm P(a',b',c)\right| \leq 2 \qquad (16)$$

5) Quantum mechanics

Quantum mechanics, however, gives certain correlations which do not satisfy the locality inequality (16).

Suppose, for example, a neutral pion is produced, by some experimental device, in some small space-time region 3. It quickly decays into a pair of photons. Suppose we have photon counters in space-time regions 1 and 2 so located with respect to 3 that when one photon falls on 1, the second falls (or nearly always does) on 2. If the π^0 is at rest the counters must be equally far away in opposite directions and their sensitive times appropriately delayed. Of course, both photons will often miss both counters. Suppose finally that both counters are behind filters which pass only photons with specified linear polarization, say at angles θ and ϕ respectively to some plane containing the axis joining the two counters.

Let us calculate according to quantum mechanics the probability of the various possible responses of the counters. If $|\theta\rangle$ denotes a photon linearly polarized at an angle θ, then for the photons going towards the counters the combined spin state is

$$|s\rangle = \frac{1}{\sqrt{2}}|0\rangle|\pi/2\rangle - \frac{1}{\sqrt{2}}|\pi/2\rangle|0\rangle \qquad (17)$$

where first and second kets in each term refer to the photons going towards regions 1 and 2, respectively. This form is dictated by considerations of parity and angular momentum. The probability that such photons pass the filters is then proportional to

$$\begin{aligned} &\frac{1}{2}\left|\langle\theta|0\rangle\langle\phi|\pi/2\rangle - \langle\theta|\pi/2\rangle\langle\phi|0\rangle\right|^2 \\ &= \frac{1}{2}\left|\cos\theta \;\sin\phi - \sin\theta\;\cos\phi\right|^2 \\ &= \frac{1}{2}\left|\sin(\theta-\phi)\right|^2 \end{aligned} \qquad (18)$$

The corresponding factor for photon 1 to pass and photon 2 not is

$$\begin{aligned} &\frac{1}{2}\left|\langle\theta|0\rangle\langle\phi+\tfrac{\pi}{2}|\tfrac{\pi}{2}\rangle - \langle\theta|\tfrac{\pi}{2}\rangle\langle\phi+\tfrac{\pi}{2}|0\rangle\right|^2 \\ &= \frac{1}{2}\left|\cos(\theta-\phi)\right|^2 \end{aligned} \qquad (19)$$

and so on. The probabilities for the various possible counting configurations are then

$$\begin{aligned}
\rho(\text{yes},\text{yes}) &= \frac{x\Omega}{4\pi}\,\frac{1}{2}\,|\sin(\theta-\phi)|^2 \\
\rho(\text{yes},\text{no}) &= \frac{x\Omega}{4\pi}\,\frac{1}{2}\,|\cos(\theta-\phi)|^2 \\
\rho(\text{no},\text{yes}) &= \frac{x\Omega}{4\pi}\,\frac{1}{2}\,|\cos(\theta-\phi)|^2 \\
\rho(\text{no},\text{no}) &= \frac{x\Omega}{4\pi}\,\frac{1}{2}\,|\sin(\theta-\phi)|^2 + x\left(1-\frac{\Omega}{4\pi}\right)+(1-x)
\end{aligned} \tag{20}$$

where x is the probability that the π^0 production mechanism actually works, Ω the (small) solid angle subtended by each counter at the production point, and no allowance has been made for bad timing, bad placing, or inefficient counting.

Now let us count $A=\pm 1$ for (yes/no) at 1 and $B=\pm 1$ for (yes/no) at 2. Then the quantum mechanical mean value of the product is

$$\begin{aligned}
P(\theta,\phi) &= \rho(\text{yes},\text{yes}) + \rho(\text{no},\text{no}) - \rho(\text{yes},\text{no}) - \rho(\text{no},\text{yes}) \\
&= 1 - \frac{x\Omega}{4\pi}\left(1+\cos 2(\theta-\phi)\right)
\end{aligned} \tag{21}$$

so that

$$\begin{aligned}
&|P(\theta,\phi) - P(\theta,\phi')| + P(\theta',\phi) + P(\theta',\phi') - 2 = \\
&\frac{x\Omega}{4\pi}\left\{|\cos 2(\theta-\phi) - \cos 2(\theta-\phi')| - \cos 2(\theta'-\phi) - \cos 2(\theta'-\phi') - 2\right\}
\end{aligned} \tag{22}$$

The right-hand side of this expression is sometimes positive. Take in particular

$$\phi = 0\ ,\quad 2\theta = \frac{\pi}{4}\ ,\quad -2\phi' = \frac{\pi}{2}\ ,\quad 2\theta' = \frac{3\pi}{4} \tag{23}$$

in which case the factor in curly brackets is

$$\{\quad\} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 2 = +2(\sqrt{2}-1) \tag{24}$$

But if quantum mechanics were embeddable in a locally causal theory (16) would apply, with $a \to \theta$, $b \to \emptyset$, and c the implicit specification of the production mechanism, held fixed in (22). The right-hand side of (22) should then be negative. So quantum mechanics is not embeddable in a locally causal theory as formulated above.

6) Experiments

These considerations have inspired a number of experiments. The accuracy of quantum mechanics on the atomic scale makes it hard to believe that it could be seriously wrong on that scale in some hitherto undiscovered way. The ground state of the helium atom, for example, is just the kind of correlated wave function which is embarrassing, and its energy comes out right to very high accuracy. But perhaps it is sensible to verify that these curious correlations persist over macroscopic distances.

Experiments so far performed do not at all approach the ideal in which the settings of the instruments are determined only while the particles are in flight. When they are decided in advance, in space time regions projecting into the overlap of the backward light cones, (16) does not follow from (12). For it was supposed in (12) that the complete specification n of the overlap is the same for the various cases compared. So one can imagine a theory which is locally causal in our sense but still manages to agree with quantum mechanics for static intruments. But it would have to contain a very clever mechanism by which the result registered by one instrument depends, after a suitable time lapse, on the setting of an arbitrarily distant instrument. So static experiments are also quite interesting.

Practical experiments are far removed from the ideal in other directions also. Geometrical and other inefficiencies lead to counters registering (no,no) with overwhelming probability, (yes,yes) very seldom, and (yes,no) and (no,yes) with probabilities only weakly dependent on the settings of the instruments. Then from (21)

$$P = 1 - \epsilon^2$$

with ϵ^2 weakly dependent on the variables, so that (16) is trivially satisfied. The authors in general make some more or less ad hoc extrapolation to connect the results of the practical with the result of the ideal experiment. It is in this sense that the entirely unauthorized "Bell's limit" sometimes plotted along with experimental points has to be understood. But such experiments also are of very high interest. For if quantum mechanics is to fail somewhere, and in the absence of a monstrous conspiracy, this should show up at some point on this side of the ideal gedanken experiment.

Several of these experiments [26),27),28)] show impressive agreement with quantum mechanics, and exclude deviations as large as might be suggested by the locality inequality. Another experiment [29)], very similar to one of those quoted [26)], is said to be in agreement with it and yet in dramatic disagreement with quantum mechanics ! And another experiment [30)] disagree significantly with the quantum prediction. Of course any such disagreement, if confirmed, is of the utmost importance, and that independently of the kind of consideration we have been making here.

7) Messages

Suppose that we are finally obliged to accept the existence of these correlations at long range, and the gross non-locality of nature in the sense of this analysis. Can we then signal faster than light ? To answer this we need at least a schematic theory of what we can do, a fragment of a theory of human beings. Suppose we can control variables like a and b above, but not those like A and B. I do not quite know what "like" means here, but suppose that beables somehow fall into two classes, "controllables" and "uncontrollables". The latter are no use for sending signals, but can be used for reception. Suppose that to A corresponds a quantum mechanical "observable", an operator $\mathcal{A}$. Then if

$$\delta \mathcal{A} / \delta b \neq 0$$

we could signal between the corresponding space time regions, using a change in b to induce a change in the expectation value of $\mathcal{A}$ or of some function of $\mathcal{A}$.

Suppose next that what we do when we change b is to change the quantum mechanical Hamiltonian $\mathcal{H}$ (say by changing some external field), so that

$$\delta \int dt \, \mathcal{H} = \mathcal{B} \, \delta b$$

where $\mathcal{B}$ is again an "observable" (i.e., an operator) localized in the region 2 of b. Then it is an exercise [31)] in quantum mechanics to show that if in a given reference system region (2) is entirely later in time than region (1)

$$\delta \mathcal{A} / \delta b = 0$$

while if the reverse is true

$$\delta \mathcal{A} / \delta b = [\mathcal{A}, -(i/\hbar) \mathcal{B}]$$

which is again zero (for spacelike separation) in quantum field theory by the usual local commutativity condition.

So if the ordinary quantum field theory is embedded in this way in a theory of beables, it implies that faster than light signalling is not possible. In this human sense relativistic quantum mechanics is locally causal.

8) Reservations and acknowledgements

Of course the assumptions leading to (16) can be challenged. Equation (22) may not embody your idea of local causality. You may feel that only the "human" version of the last section is sensible and may see some way to make it more precise.

The space time structure has been taken as given here. How then about gravitation ?

It has been assumed that the settings of instruments are in some sense free variables - say at the whim of experimenters - or in any case not determined in the overlap of the backward light cones. Indeed without such freedom I would not know how to formulate any idea of local causality, even the modest human one.

This paper has been an attempt to be rather explicit and general about the notion of locality, along lines only hinted at in previous publications [Refs. 2), 4), 10), 19)]. As regards the literature on the subject, I am particularly conscious of having profited from the paper of Clauser, Horne, Holt and Shimony [3], which gave the prototype of (16), and from that of Clauser and Horne [16]. As well as a general analysis of the topic this last paper contains a valuable discussion of how best to apply the inequality in practice; I am indebted to it in particular for the point that in two-body decays (as compared with three-) the basic geometrical inefficiencies enter in (22) in a relatively harmless way. I have also profited from many discussions of the whole subject with Professor B. d'Espagnat.

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