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Threshold autoregression in economics

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The impact of Howell Tong's threshold autoregressive (TAR) model in the fields of econometrics and economics is documented by a review of the enormous literature.

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1. INTRODUCTION

The threshold autoregressive (TAR) model developed by Howell Tong has been enormously influential in economics. A simple illustration of the depth of this influence is to examine the recent edition of the graduate text *Applied Econometric Time Series* by Walter Enders [25] and observe that 60 pages of the text are devoted to nonlinear time series models, with the bulk devoted to TAR models.

My goal in this paper is to document the influence of Howell Tong's TAR model in the fields of econometrics and economics. I do so by reviewing 75 papers published in the econometrics and economics literatures which either contribute to the theory of TAR estimation and inference, or report a substantive application of a TAR model to economics. Many of these papers are themselves highly cited. This literature is enormous, and the papers reviewed here are not an exhaustive list of all applications of the TAR model. I focus on the more substantial and influential papers.

Closely related to the TAR model is the smooth-transition autoregressive (STAR) model which has been equally influential in economics. To keep the attention focused, I do not review this literature here. Extensive reviews on the STAR model in economics can be found in the monographs by Granger and Terasvirta [38] and Franses and van Dijk [31], and the review paper by van Dijk, Terasvirta and Franses [23].

In Section 2 I review the literature on the econometric theory of TAR estimation and inference. In Section 3 I review empirical applications of the TAR model.

2. TAR MODELS IN ECONOMETRIC THEORY

There has been been a substantial number of papers written in econometric theory concerning estimation and inference methods for the TAR model.

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As described in Section 5.3 of Tong's review [76], one statistical issue of primary importance is testing for linearity against the TAR alternative. Such tests are of particular importance in economics as there is a presumption among most economists that applications should use linear models unless there is convincing evidence to support a specific nonlinear specification. Tests for a threshold effect provide such evidence. Hansen [40] is a general investigation of testing in the context of unidentified nuisance parameters, and proposed a feasible yet rigorous simulation method to calculate critical values and p-values for such tests. The paper details implementation of this testing method for TAR models. The methods are explored in further detail in Hansen [43]. These methods were extended to models with multiple thresholds by Gonzalo and Pitarakis [32]. Pitarakis [64] investigated the impact of autoregressive lag length selection when conducting the test for a threshold and developed an asymptotic theory of estimated lag lengths in misspecified linear models.

Tong [76] also raises in Section 5.2 the issue of inference on the threshold parameter. This is necessary to construct confidence intervals for the threshold parameter. Hansen [44] found that a nuisance parameter-free asymptotic approximation can be developed for tests on the threshold parameter by modeling the threshold effect as small (decreasing with sample size). This yields a computationally attractive method for constructing confidence intervals, and is described in detail in Hansen [41]. This method was extended to panel data models by Hansen [42], to models with multiple thresholds by Gonzalo and Pitarakis [32], and to models with endogenous regressors by Caner and Hansen [18]. To improve on Hansen's asymptotic approximation, Gonzalo and Wolf [34] propose a subsampling procedure.

The least-squares estimate of the threshold has a nonstandard asymptotic distribution. As an alternative, Seo and Linton [74] propose a smoothed least squares estimator which achieves an asymptotic normal distribution and produces confidence intervals with excellent coverage performance. The Seo-Linton estimator also allows the thresholding to depend on a linear index of observed variables. Caner [16] shows that median threshold regressions can be estimated by least absolute deviations (LAD), and that the threshold estimate has a similar asymptotic distribution as the least-squares estimator under the small-threshold-effect asymptotic structure of Hansen [44].

Economic time series are widely believed to be nonstationary. An issue of critical importance in applied economics is to be able to distinguish nonlinear times series from nonstationary series. In a TAR model, these questions lead to two important classes of statistical tests: testing the hypothesis of a unit root (linear nonstationarity) against a stationary TAR (nonlinear stationary); and testing the hypothesis of linearity against the alternative of nonlinearity while allowing for the possibility of nonstationarity under the null hypothesis. A series of papers in the econometrics literature have addressed these issues. Enders and Granger [27] considered the problem of testing the unit root null when the threshold parameter is known. Tests allowing for estimated thresholds have been explored by Caner and Hansen [17], Lanne and Saikkonen [52], Kapetanios and Shin [47], Pitarakis [65], Bec, Guay and Guerre [12] and Seo [72]. There are notable differences between these papers. Caner and Hansen, for example, require that the threshold variable is stationary while Seo allows the threshold variable to be nonstationary under the null hypothesis. Caner and Hansen [17] also develop an asymptotic test of the hypothesis of a unit root process against the alternative of a stationary TAR model.

Balke and Fomby [7] introduced a "threshold cointegration" multivariate model which allows a discontinuous adjustment to long-run equilibrium. It is a combination of Tong's TAR model and Engle and Granger's model of cointegration [30] known as a vector error-correction model (VECM). The threshold cointegration model has been enormously influential in economics. Tsay [77] introduced a similar vector TAR (VTAR) model with possible threshold cointegration. Enders and Siklos [28] extended the Engle-Granger test for cointegration to allow for threshold adjustment. Estimation methods for the threshold VECM model were developed by Hansen and Seo [45], who also develop tests for the presence of threshold effects within the threshold VECM. Gonzalo and Pitarakis [33] develop similar tests in the context of a single cointegrating regression. Seo [71] develops a test for the null of no cointegration against the alternative of threshold cointegration. Seo [73] develops an asymptotic distribution theory for the estimates of the cointegration vector in the threshold VECM model.

3. TAR MODELS IN EMPIRICAL ECONOMICS

3.1 Output growth

One major application of nonlinear time series analysis in economics has been to the business cycle. In a common application, TAR models have been used to model aggregate output as measured by GNP growth rates. An early and influential paper in this literature is Beaudry and Koop [11] who model GNP growth rates as a function of the deviation of the current level of GNP from its historical maximum. They report evidence supporting the asymmetric effect of deviations from past output on future economic growth. Pesaran and Potter [62] show that the model in Beaudry and Koop [11] is a special case of a TAR model, and Pesaran-Potter use the TAR framework to to show that US GNP is subject to floor and ceiling effects. Another influential paper applying Tong's SETAR model to US GNP growth rates is Potter [66]. Hansen [40] uses Potter's example to demonstrate the relevance of statistical tests of linear autoregressions against TARs. Koop and Potter [49] estimate TAR models for U.S. unemployment using Bayesian methods and find strong evidence for nonlinearity. Enders, Falk and Siklos [26] estimate a TAR model for US GDP and use the estimates to illustrate methods for constructing confidence intervals for the model parameters. Peel and Speight [60] present evidence that SETAR models are appropriate for output in a set of industrialized economies.

A second generation of applications used multivariate models, in particular the threshold VECM and VTAR models, to jointly model aggregate output and other variables. Koop, Pesaran and Potter [48] develop methods for impulse response analysis in nonlinear multivariate models and apply these methods to a US output and the unemployment rate using a multivariate extension of Beaudry and Koop [11]. Balke [6] uses a TVAR model for output growth, inflation, the Fed funds rate and a measure of credit market conditions. He finds that shocks have a larger impact on output when the credit measure is above a threshold, indicating that "tight" credit regimes multiply shocks more than normal regimes. Altissimo and Violante [3] use a VTAR to study the joint dynamics of U.S. output and unemployment rate. They find that only the unemployment rate reacts nonlinearly, transmitting to output through cross-correlation.

3.2 Forecasting

A major application of TAR models has been to the forecasting of economic variables. The common goal has been to compare the relative forecasting performance of TAR and other nonlinear models with traditional linear (or even random walk) forecasts. In this literature, the models are typically compared using out-of-sample mean-square forecast error. The overall evidence is mixed. Some authors find improvements using TAR models, other find no clear improvements. Rothman [68] compares the forecasting performance of a set of nonlinear autoregressions for the U.S. unemployment rate. He finds the best performance is obtained by an EAR model and a generalized autoregression based on a Volterra expansion. Montgomery, et al. [57] also compare the empirical forecasting performance of a set of time-series models for the U.S. unemployment rate. They find significant reductions in mean-squared forecast error by TARs relative to linear models during periods of contraction. Clements and Smith [21] examine SETAR models for forecasting U.S. GNP and find that the relative performance of SETAR models varies with the sample period and the conditioning information. Clements and Smith [22] examine the performance of SETAR models for forecast densities of U.S. output growth and changes in the unemployment rate.

Brooks [15] examines the ability of TAR and other nonlinear models to improve forecasts of exchange rates relative to linear models. He founds modest improvements from the use of nonlinear models.

3.3 Interest rates

Several authors have used multivariate TAR models to model the term structure of interest rates (the relationship between long and short interest rates). The common finding is that there is a strong asymmetric response of interest rate changes to the spread between the long and short rates. Papers in this literature include Pfann, Schotman and Tschernig [63] who use a TAR model with conditional heteroskedasticity, Anderson [4] who uses a smooth transition threshold VECM, Tsay [77] who uses a VTAR model, and Hansen and Seo [45] who used a threshold VECM.

In related work, Baum and Karasulu [10] use a threshold VECM to model the federal reserve discount rate policy, and Sander and Kleimeier [69] use a threshold VECM to investigate the pass-through of monetary policy (interest rate changes) into the market sector. Gospodinov [36] finds threshold effects in both the autoregressive and GARCH errors for interest rates.

3.4 Prices

A common theme in the application of TAR models to economic price data has been transaction costs. Economic arbitrage requires that the prices of related goods move together, but the presence of transaction costs can produce a band-threshold effect, where only deviations above a threshold will have an effect on price movements.

Yadav, Pope and Paudyal [79] use a univariate TAR model for the difference between the price of an index futures contract and the equivalent underlying cash index and argue that this is consistent with arbitrage-related transaction costs.

Other applications have been primarily multivariate. Martins, Kofman, and Vorst [55] use a band-TAR model to account for transaction costs on futures prices. Combining the threshold structure with an error-correction model, they find asymmetry in the reaction of index returns to lagged futures returns. Lo and Zivot [54] is an extensive application of the threshold VECM methodology to a large set of U.S. disaggregated price data, and find evidence of threshold cointegration for tradable goods. Huang, Hwang and Peng [46] use a VTAR model to investigate the impact of oil price changes on economic activity. Al-Gudhea, Kenc, and Dibooglu [2] use a threshold VECM to investigate the relation between daily wholesale and retail gasoline prices. They find asymmetric responses, in that increases in wholesale prices are passed to retail prices more quickly than decreases in wholesale prices.

TAR models have also been quite influential in the study of asymmetric price transmission in agricultural economics. For example, applications to the price of pork can be found in Goodwin and Harper [35] and Absulai [1]. A broad survey of this literature is Meyer and von Cramon-Taubadel [56].

3.5 Stock returns

An important class of economic time-series models are the ARCH family of conditionally heteroskedastic processes introduced by Engle [29] and widely used to handle financial time-series. Cao and Tsay [19] applied TAR processes with ARCH to U.S. stock indices. Rabemananjara and Zakoian [67] introduced a threshold effect into the ARCH equation, allowing nonlinearity in the volatility process. They applied the model to French stock indices. Li and Li [53] extend their model to a “double threshold” ARCH model, and applied the method to the daily Hong Kong Hang Seng Index.

Domian and Louton [24] use a TAR model to find a pronounced threshold asymmetry in the relation between stock returns and real economic activity. Barnes [9] uses a VTAR to investigate the relation between nominal returns and inflation.

In recent applications, Narayan [58] applies an unrestricted TAR model to monthly stock price indices and finds evidence for a TAR model with a unit root, and Griffin, Nardari and Stulz [39] use a VTAR to investigate the dynamic relation between stock market turnover, volatility and returns across 46 countries.

3.6 Exchange rates

There has been much literature applying TAR models to exchange rates. In an early application, Krager and Kugler [50] use SETAR models for five foreign exchange series, finding statistically significant threshold effects. Chappel, Padmore, Mistry and Ellis [20] use a SETAR to the French franc/deutschmark exchange rate, and find improved fit and superior forecasting performance relative to a linear random walk. Balke and Wohar [8] use a TAR model for deviations from covered interest parity. They interpret the TAR model as evidence for asymmetric transaction costs. O’Connell [59] applies Balke-Fomby’s threshold cointegration model to a set of real exchange rates to investigate the theory of purchasing power parity (PPP) under transaction costs. In an influential paper, Taylor [75] uses a TAR model to demonstrate that linear autoregressions can give very misleading estimates of persistence and mean-reversion.

Peel and Taylor [61] use a TAR model to model covered interest arbitrage during the interwar period. A broad-based empirical investigation of PPP across exchange rates and sectors using TAR models was done by Sarno, Taylor and Chowdhury [70]. Bec, Salem and Carrasco [14] show that a set of European exchange rates reject the null hypothesis of a linear unit root process in favor of the alternative that the series are stationary three-regime SETAR models. Gouveia and Rodrigues [37] use SETAR models and threshold VECM to find evidence of an asymmetric adjustment in a relative version of PPP for eight exchange rate pairings. Kapetanios and Shin [47] propose a test to distinguish a unit root process from a stationary three-regime SETAR process, and apply the test to G7 exchange rates, finding evidence in favor of the SETAR models. Juvenal and Taylor [51] find evidence of nonlinearity in the law of one price using SETAR models for 16 sectors in nine European countries.

Aslanidis and Kouretas [5] use a threshold VECM to model the relationship between the parallel and official markets for US dollars in Greece.

TAR models have even been applied in economic history. Volckart and Wolf [78] use a TAR to assess medieval financial integration between European regions in the 14th and 15th centuries.

4. CONCLUSION

As this review documents, econometrics and economics have been greatly influenced by the TAR model. Consequently, we owe a debt to Howell Tong for his visionary innovation.

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