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Author(s): Ole Peters, Anna Deluca, Álvaro Corral, J. David Neelin, and Chris E. Holloway.  
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# Universality of rain event size distributions

Anna Deluca<sup>1\*</sup>, O. Peters<sup>2,3</sup>, A. Corral<sup>1</sup>, J. D. Neelin<sup>3</sup>, C. E. Holloway<sup>4</sup>

<sup>1</sup>Centre de Recerca Matemàtica, Spain

<sup>2</sup>Dept. of Mathematics and Grantham Institute for Climate Change, Imperial College London, UK.

<sup>3</sup>Dept. of Atmospheric and Oceanic Sciences and Institute for Geophysics and Planetary Physics, UCLA, USA

<sup>4</sup>Dept. of Meteorology, University of Reading, UK

Atmospheric convection and precipitation have been hypothesised to be a real-world realization of self-organized criticality (SOC). This idea is supported by observations of avalanche-like rainfall events<sup>1,2</sup> and by the nature of the transition to convection in the atmosphere<sup>4,5</sup>. However, many questions remain open. Here we ask whether the observation of scale-free avalanche size distributions is reproducible using data from different locations and whether the associated fitted exponents show any sign of universality.

We study rain data from all 10 available sites of the Atmospheric Radiation Measurement (ARM) Program, see [www.arm.gov](http://www.arm.gov), over periods from about 8 months to 4 years<sup>3</sup>. The measurements are from climatically different regions using a standardized technique, making them ideal for our purpose. Precipitation rates were recorded at one-minute resolution with an optical rain gauge.

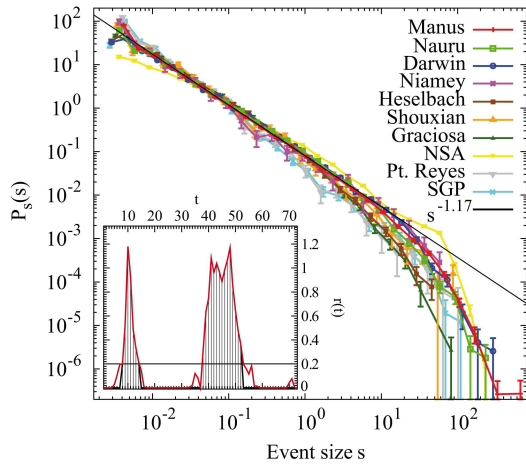


Figure 1. Probability densities of event sizes,  $s$  in mm, and a power-law fit (black straight line). Inset: Precipitation rates including two rain events lasting 7 and 15 minutes respectively. Interpreting reported rain rates of less than 0.2 mm/h as zero, the shaded areas are the corresponding event sizes.

Following Ref.<sup>2</sup>, we define an event as a sequence of non-zero measurements of the rain rate, see inset in the figure. The event size  $s$  is the rain rate,  $r(t)$ , integrated over the event,  $s = \int_{\text{event}} dt r(t)$ . For each data set, the probability density function  $P_s(s)$  in a particular size interval  $[s, s + \Delta s)$  is estimated.

The distributions, shown in the figure, are visually compatible with a power law (black straight line) over most of their ranges. A procedure similar to that in Ref.<sup>8</sup> consisting of maximum-likelihood estimation plus a goodness-of-fit test confirms this result: over ranges

between 2 and 4 orders of magnitude, all data sets are consistent with a power-law distribution and the estimates of the apparent exponents are in agreement with the hypothesis of a single exponent  $\tau_s = 1.17(3)$ , except for three problematic data sets from Point Reyes, the Southern Great Plains and Alaska.

Climatic differences between regions are scarcely detectable in event size distributions, which may be surprising on the grounds of climatological considerations. However, the cutoff  $s_\xi$ , representing the capacity of the climatic region around a measuring site to generate rain events, changes significantly from region to region, confirming meteorological intuition, and is easily extracted from the moments of the distributions. While the exponents are not significantly different, the larger tropical events are reflected in the greater large-scale cutoff of the tropical distributions.

Similarly, the dry-spell durations (durations of precipitation-free intervals) seem to follow another power law with  $\tau_d = 1.2(1)$ , and regional differences can be seen in the strength of the diurnal cycle and the cutoff dry spell duration. The broad range of event durations suggests a link to the lack of characteristic scales in the mesoscale regime, where approximately scale-free distributions of clusters of convective activity, for example cloud or precipitation, have been observed to span areas between  $\mathcal{O}(1 \text{ km}^2)$  and  $\mathcal{O}(10^6 \text{ km}^2)$ , see for example Ref.<sup>9</sup>. The observation of scale-free rainfall event sizes suggests long-range correlation in the pertinent fields, a possible indication of critical behaviour near the transition to convective activity.

\* [adeluca@crm.cat](mailto:adeluca@crm.cat)

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